Chapter-14 Black Body Radiation

* **Radiation**: The transfer of heat energy in the vaccum in the form of electromagnetic wave is known as radiation. or, The process of transformation of heat without disturbing the medium is called radiation and transferred heat energy by the radiation is called radiant heat or thermal radiation. It is also called infrared radiation.

* **Total energy density (u)**: It is defined as the energy per unit volume due to all possible wavelengths of the radiation. It is denoted by u and its unit is J/m^3.

* **Spectral energy density (u_λ)**: The spectral energy density is defined as the energy per unit volume per unit wavelength range of thermal radiation. It is denoted by u_λ. Therefore u and u_λ are related as $u = \int_0^{+\infty} u_\lambda d\lambda$.

* **Total emmisive power (E)**: It is defined as the emitted thermal radiation (energy) per unit area per unit time. It is denoted by E.

* **Spectral emmisive power (E_λ)**: It is defined as the thermal energy emitted per unit area

per unit time. It is denoted by E_λ.
Therefore, $E = \int_0^\infty E_\lambda d\lambda$

* Absorptive power (a_λ): It is defined as the ratio of the absorbed thermal radiation per unit area per unit time. It is denoted by a_λ.

(I) * Kirchhoff's law: This law states that the ratio of emmisive power to the absorptive power at give temperature and wave length is the same for all bodies and is equal to the emmisive power of a black body at that temperature.

i.e. $\frac{e_\lambda}{a_\lambda} = E_\lambda$ = constant.

Proof:
Consider a body is placed in an isothermal enclosure and dQ be the amount of radiant energy per unit area per unit time on the body. If a_λ be the absorptive power of the body, the amount of energy absorbed by the body per unit area per unit time is $a_\lambda dQ$. Then the remaining portion of energy i.e. $(1-a_\lambda) dQ$ is either reflected or transmi

tted from the body

Similarly, e_λ be the emissive power of the body in the wavelength range λ to $\lambda + d\lambda$, the amount of energy emitted by the body per unit area per unit time is $e_\lambda d\lambda$

As the system is in isothermal condition, so amount of heat incident = amount of heat transmitted.

i.e. $dQ = dQ(1 - a_\lambda) + e_\lambda d\lambda$

or, $dQ = dQ - a_\lambda dQ + e_\lambda d\lambda$

or, $e_\lambda d\lambda = a_\lambda dQ$

or, $\frac{e_\lambda}{a_\lambda} = \frac{dQ}{d\lambda} \longrightarrow (I)$

This shows that the ratio of emmissive power to the absorptive power is constant for all bodies.

$\therefore \frac{e_\lambda}{a_\lambda} = \text{constant} \longrightarrow (II)$

For black bodies,

$a_\lambda \approx 1$ & $e_\lambda \approx \text{maximum} = E_\lambda$ (say)

Then eqn. (I) becomes.

$E_\lambda = \frac{dQ}{dt} \longrightarrow (III)$

Pressure of radiation

The radiation posses the properties of light, so like light, it exarts small but definite pressure on the surface on which it is incident called pressure of radiation.

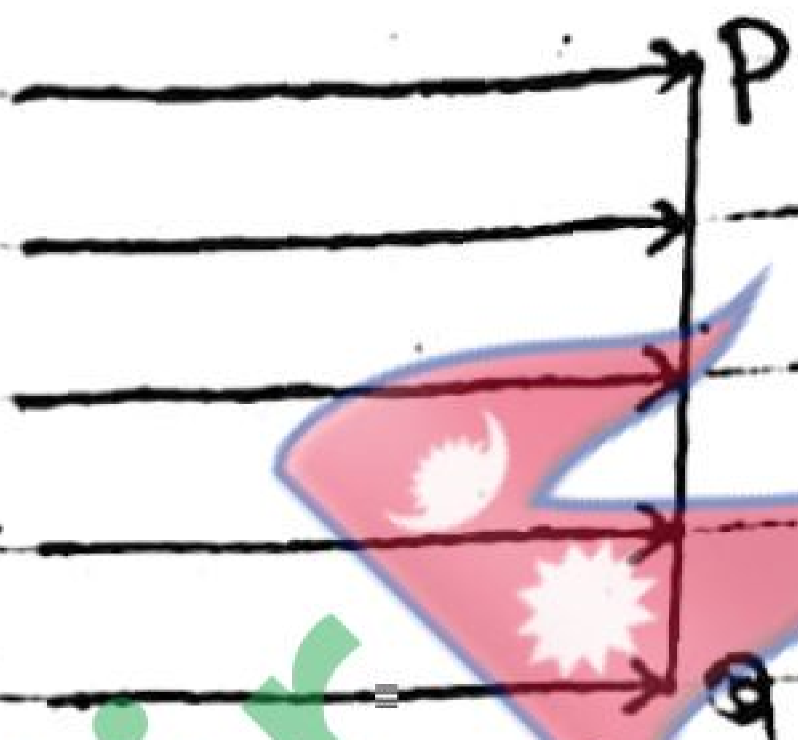

Fig.: pressure of radiation

Let a photon of energy $h\nu$ move with velocity of light c. According to the theory of relativity,

$$h\nu = mc^2, \quad \nu = \text{frequency}$$

$$m = \frac{h\nu}{c^2}$$

Then, momentum (p) = mass (m) × speed (c)

$$= \frac{h\nu}{c^2} \times c$$

$$= \frac{h\nu}{c} = \frac{e}{c} \quad e = \text{emissivity}$$

Now, Momentum incident on surface PQ per unit area per unit time is given by

$$p = \frac{\Sigma e}{c} = \frac{E}{c}$$

If u is the energy density then total energy passing through any area of surface

normal to radiation per time,
$E = u \times S \times c$, S = area

Energy flux (E) $= \frac{uSc}{S} = uc$
Then, $p = \frac{uc}{c} = u$.

which shows that the pressure of radiation is equal to the energy density.

* Pressure of diffuse radiation

Let us consider energy falling on QR plane making an angle θ with its axis. PR is another plane which make an angle θ with QR. Let u be the energy density and S be the surface area of PQ where as surface area of QR is S'.

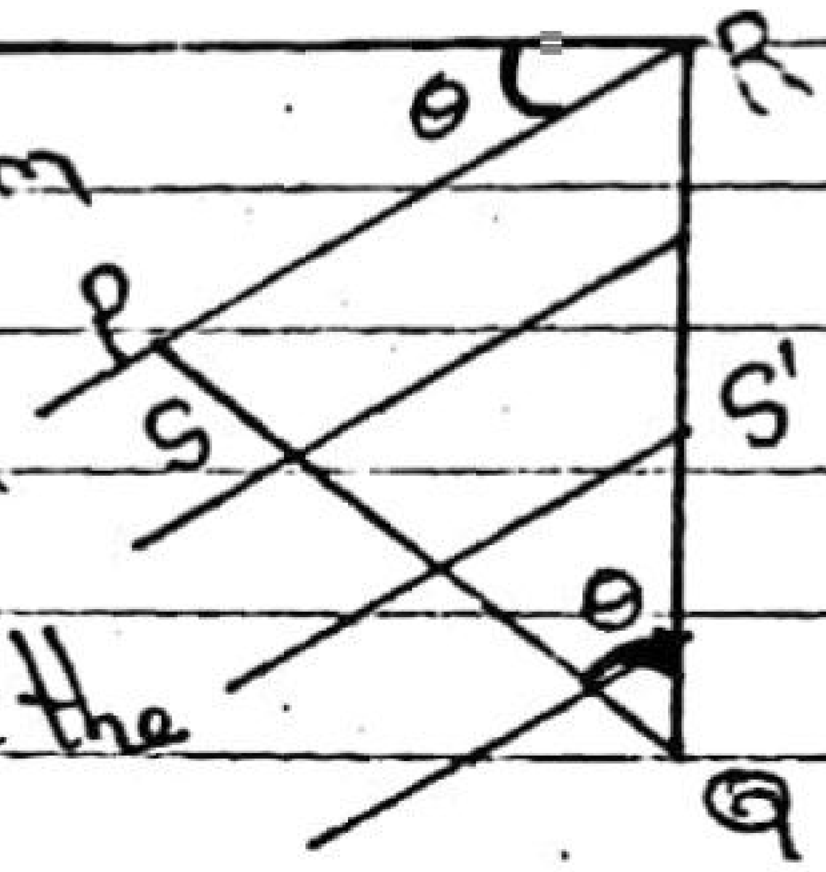

Fig: pressure of diffuse radiation

We have,

$$\text{Pressure radiation}(p) = \frac{\text{energy per unit area per unit time}}{\text{speed of light}}$$

Here,

Energy on plane PR per second $= uSc$

from fig. $\cos\theta = \frac{S}{S'}$

$S = S' \cos\theta$

Therefore, Energy on plane per second

$= u S' \cos\theta \, c$

$\therefore$ The amount of energy crossing PR plane is equal to energy incident on QR

Energy per unit area per unit time on plane

$$QR = \frac{u S' \cos\theta c}{S'}$$

$$= uc\cos\theta$$

So, pressure of diffused radiation $(P) = \frac{u\cos\theta c}{c}$

$$\therefore P = u\cos\theta \longrightarrow (I)$$

If all of this momentum is absorbed then eqn (I) gives pressure of radiation.

Here, normal component of pressure due to diffuse radiation is

$$P_n = P\cos\theta$$

$$= u\cos\theta \cdot \cos\theta$$

$$\therefore P_n = u\cos^2\theta$$

or, $\langle P_n \rangle = \langle u\cos^2\theta \rangle$

But $\langle \cos^2\theta \rangle = 1/3$

Then,

$$\langle P_n \rangle = \langle \frac{u}{3} \rangle$$

$\therefore$ pressure of diffuse radiation $= \frac{u}{3}$

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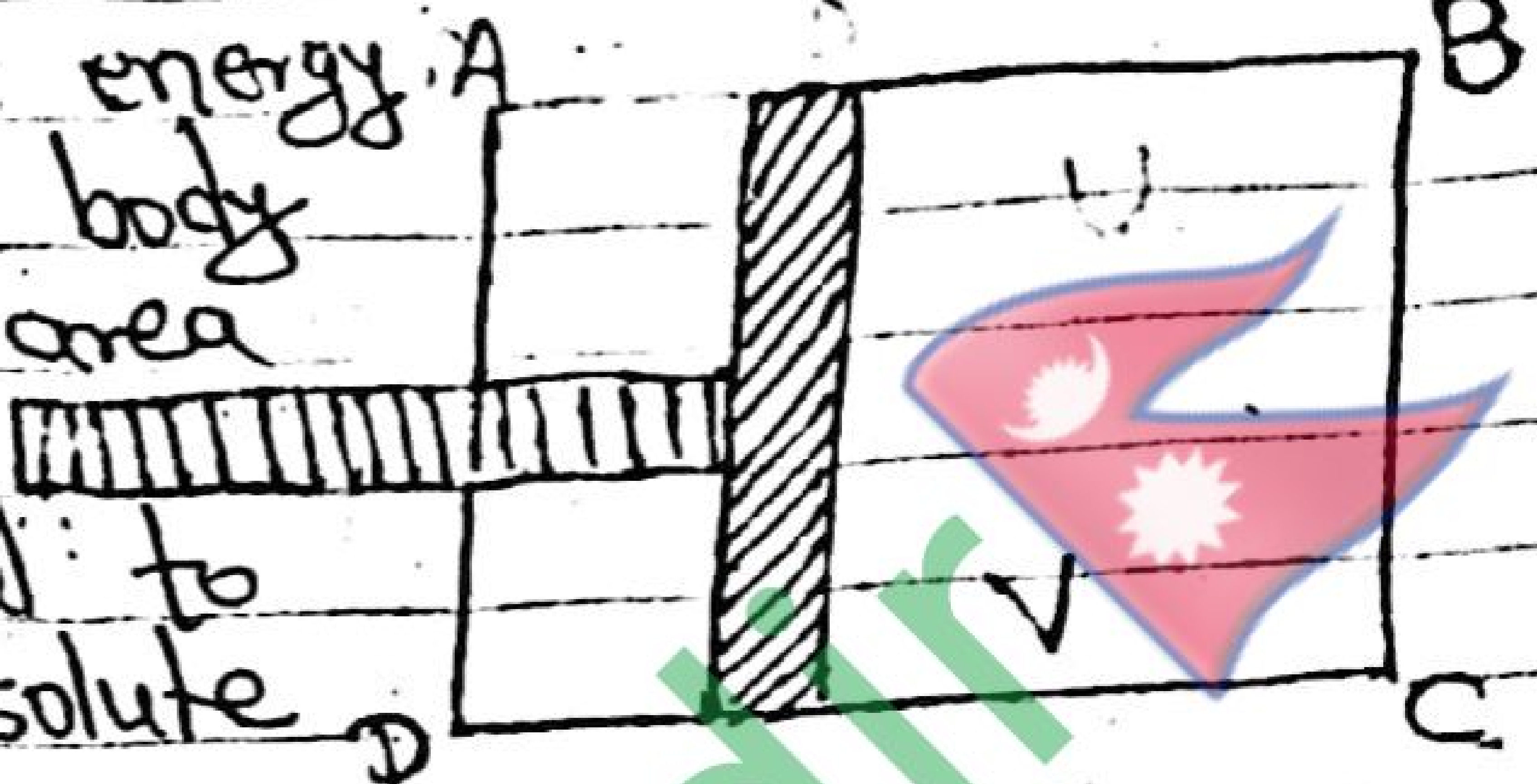
* Stefan's - Boltzmann's law



"It states that energy radiated by black body per unit surface area per second is directly proportional to 4^{th} power of absolute temperature."

i.e. $E \propto T^4$

$E = \sigma T^4$

Where σ be the stefan's constant and its value is $5.67 \times 10^{-8} W/m^2K^4$. Suppose a cylinder ABCD provided with perfectly reflecting wall and piston P, u be the energy density of diffuse radiation inside it in volume V, then

Total energy $(U) = uV \longrightarrow (I)$

From 1^{st} law of thermodynamics;

$dq = dU + dW$

$\therefore dq = d(uV) + PdV \longrightarrow (2)$

From 2^{nd} law of thermodynamics.

$dq = dST \longrightarrow (3)$

Also, pressure radiation $(P) = \frac{u}{3} \longrightarrow (4)$

From eqn. (2) & (3),

$Tds = d(uV) + Pdv$

Or, $Tds = udv + vdu + pdv$

Or, $Tds = vdu + udv + \frac{4}{3}dv$

Or, $Tds = vdu + \frac{4}{3}udv$

$\therefore ds = \frac{vdu}{T} + \frac{4}{3}\frac{udv}{T} \longrightarrow (5)$

Here, s be a function of u and v

so, $s = f(u,v)$

$ds = \frac{\partial s}{\partial u}du + \frac{\partial s}{\partial v}dv \longrightarrow (6)$

Comparing (5) & 6, we have

$\frac{\partial s}{\partial u} = \frac{v}{T}$

$\frac{\partial s}{\partial v} = \frac{4}{3}\frac{u}{T}$

As ds is perfect differential, we can write,

$\frac{\partial^2 s}{\partial u \partial v} = \frac{\partial^2 s}{\partial v \partial u}$

or, $\frac{\partial}{\partial u}\left(\frac{\partial s}{\partial v}\right) = \frac{\partial}{\partial v}\left(\frac{\partial s}{\partial u}\right)$

Or, $\frac{\partial}{\partial u}\left(\frac{4}{3}\frac{u}{T}\right) = \frac{\partial}{\partial v}\left(\frac{v}{T}\right)$

Or, $\frac{4}{3T}\frac{\partial u}{\partial u} + \frac{4}{3}\frac{(-1)\,\partial T}{T^2\,\partial u} = \frac{1}{T}$

or, $\frac{4}{3T} - \frac{4u\,\partial T}{3T^2\,\partial u} = \frac{1}{T}$

or, $\frac{1}{3T} = \frac{4u}{3T^2}\frac{\partial T}{\partial u}$

or, $\frac{\partial T}{\partial u} = \frac{3T}{4u \cdot 3}$ or, $\frac{\partial T}{T} = \frac{1}{4}\frac{\partial u}{u}$

or, $\frac{\partial u}{u} = 4\frac{\partial T}{T}$

Integrating,

$\int\frac{\partial u}{u} = 4\int\frac{\partial T}{T}$ or $\Rightarrow \ln u = 4\ln T + \ln A$

or, $\ln u = \ln T^4 + \ln A$, or; $\ln u = \ln AT^4$

or, $u = AT^4$

But $E = uc/4$ then

$E = \frac{Ac}{4}T^4$

$E = \sigma T^4 \longrightarrow (7)$

Where $\sigma = \frac{Ac}{4}$

which is the required eq^n of stefan's boltzman law.

plank's radiation law

plank derive a formula to explain the experimental observation of the distribution of energy in black body radiation. The formula was derived using the following postulates:

1. A black body radiation chamber is filled up not only with radiation but also with simple harmonic oscillator which can have energy given by $E = nh\nu$, where, ν = frequency of oscillator, h = plank constant, $n = 0, 1, 2, 3$ ---
2. The average energy of the oscillators in the black body radiation is given as;

$$E = \frac{h\nu}{e^{\frac{h\nu}{k_BT}} - 1} \longrightarrow (1)$$

3. The no. of oscillator in the frequency range ν to $\nu + d\nu$ is given by

$$N_\nu d\nu = \frac{8\pi\nu^2 d\nu}{C^3} \longrightarrow (2)$$

Since, equation (1) gives the energy of each ocillators and equation (2) gives the no. of oscillators per unit volume, the energy density of black body radiation in the frequency range ν to $\nu + d\nu$ is

$$EN_\nu d\nu = \frac{h\nu}{e^{\frac{h\nu}{K_BT}} - 1} \times \frac{8\pi\nu^2 d\nu}{c^3} \longrightarrow (3)$$

Since $\nu = c/\lambda \Rightarrow |d\nu| = \frac{c}{\lambda^2} d\lambda$

Then eq^n (3) reduces the form

$$EN_\lambda d_\lambda = \frac{8\pi hc}{\lambda^5 \left(e^{\frac{hc}{\lambda K_BT}} - 1\right)} d\lambda \longrightarrow (4)$$

This relation is known as plank law of radiation

Wien's Displacement law

It states that the product of the wavelength corresponding to maximum energy 'λ_m' and absolute temperature 'T' is constant i.e. $\lambda_m T$ = cons. This constant is called wien's displacement ~~law~~ constant and its value is 2.896×10^{-3} mK

Wien's law and Rayleigh-Jeans law from plank's law of radiation

→ For short wavelength range :-

If the wavelength of the radiant energy is shorter, the quantity $e^{\frac{hc}{\lambda K_BT}}$ dominates the unity and hence equation (4) becomes,

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{-hc}{e^{\frac{-hc}{\lambda K_B T}}} d\lambda \longrightarrow (5)$$

Equation (5) represents the energy density distribution of the radiant energy in the shorter wavelength range and this eq^n is called wein's modification to plank's law of radiation.

→ For large wavelength:-

If the wavelength of the radient energy is larger, the term $e^{\frac{hc}{\lambda K_B T}}$ in eq^n (4) can be ~~explained~~ expanded as

$$e^{\frac{hc}{\lambda K_B T}} = \left(1 + \frac{hc}{\lambda K_B T} + \ldots\right) \longrightarrow (6)$$

(neglecting higher power of small quantity)

Using eq^n (6) in eq^n (4), the plank law of radiation modifies to the form,

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 \left[1 + \frac{hc}{\lambda K_B T} - - - 1\right]} d\lambda$$

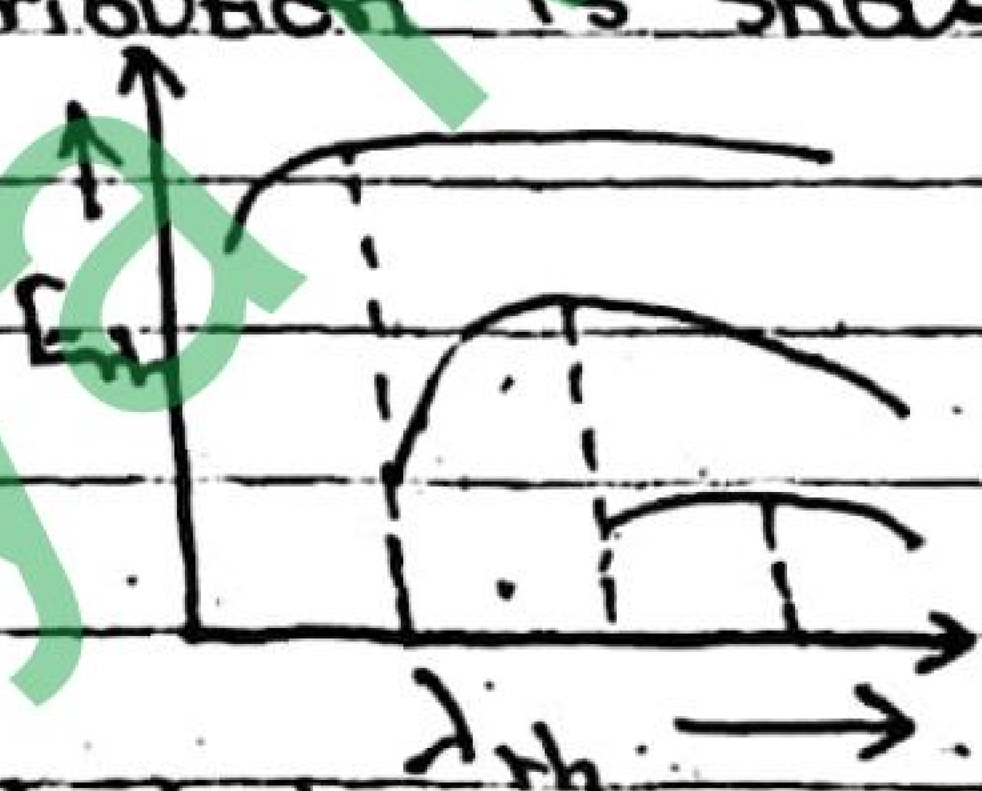
$$= \frac{8\pi hc}{\lambda^5} \times \frac{K_B T}{hc} \lambda d\lambda$$

$$E_\lambda d\lambda = \frac{8\pi K_B T}{\lambda^4} d\lambda \longrightarrow (7)$$

Equation (7) represents the energy density distribution of radiant energy in the long wavelength range. This modification of plank's law is Rayleigh-Jean's modification or Rayleigh Jean's law of radiations.

Energy distribution in the spectrum of black body radiation.

⇒ Lummer and his coworker performed an experiment to study the distribution of energy in spectrum of black body radiation. They performed the experiment by heating a black body at different temperature and study the variation of energy with wavelength of the radiation. The observed distribution is shown in following plot.



From this distribution curve, following conclusion can be drawn

1. The energy of black body radiation is not continuous.
2. The intensity of black body radiation at a

temperature increases with increase in wavelength become maximum and then decreases on further increase in wavelength.

3) The value of λ_m goes on decreasing order on increasing the temperature which λ_m is the value of wavelength at which black-body radiation has maximum intensity.

4) The intensity of black body radiation increases with increase in temperature.

5) The area of each curve is proportional to the 4^{th} power of absolute temperature T. Therefore the area under each curve represents the energy i.e. $E = \sigma T^4$. (Stefan's law)

Plank's hypothesis of black body radiation (Quantum theory of radiation)

⇒ To explain the observed distribution of energy in the spectrum of black body radiation, plank in 1900 A.D. gave a hypothesis by defining an empirical constants (called Plank's constant) as follows:

" A black-body chamber is not only filled with the radiations of all possible wavelength also the simple harmonic oscillators of molecular diameter dimension such that the exchange of energy between matter and radiation is discontinuous and discrete in the form of some integral multiple of some unit of energy called quantum".

That means the energy of black-body radiation is given by $E = nh\nu$ where,

$n = 0, 1, 2, 3 \ldots$

h = plank's constant

ν = frequency of oscillator

This hypothesis was later used by Einstein to explain the photo electric effect and specific heat of solid and liquids. Similarly Bohr also uses the hypothesis to explain the atomic spectrum.

Solar constant (Estimation of temp. of sun)

Solar constant is defined as the amount of solar solar energy received per minute by unit area of a perfectly black body placed at a mean distance of the earth from from the sun, in the absence of the atmosphere and at right angle to the

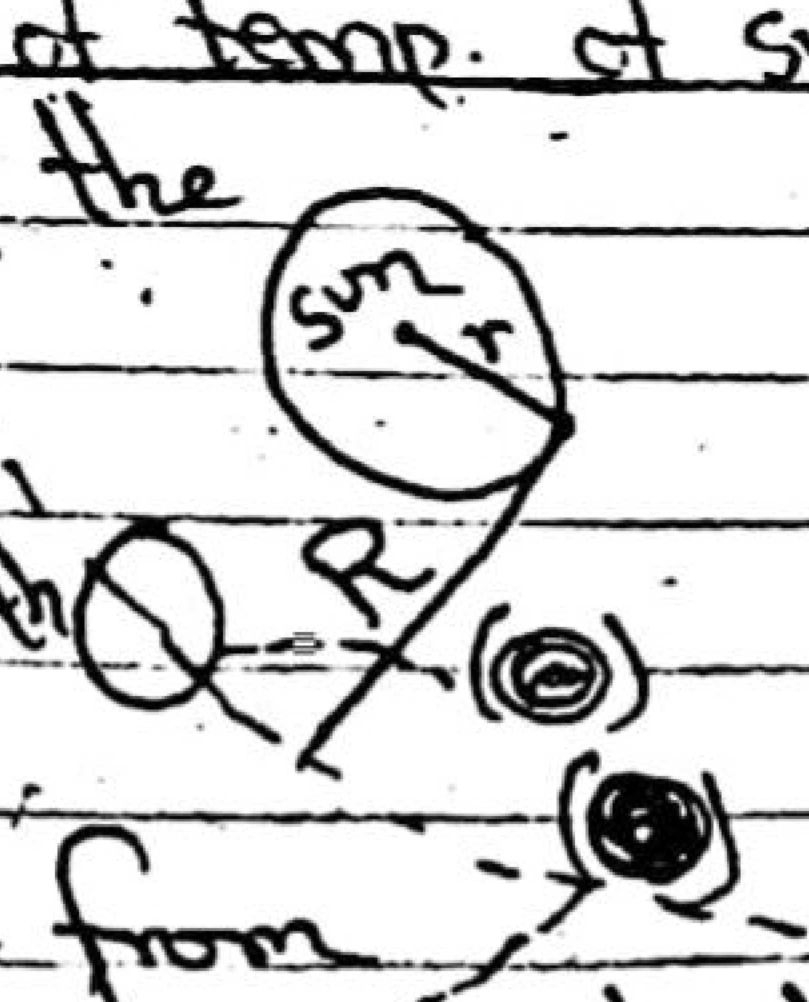


fig: Solar constant

direction of sun rays. It is denoted by S.
The concept of solar constant is used to estimate the temp^r of sun and stars. Let E be the emissive power of sun, r be the its radius. Then the radiant energy emitted by the sun per minute =

$$4\pi r^2 E \times 60 \longrightarrow (1)$$

Again, S be the solar constant, R be the mean distance between sun and earth then the radiant energy absorbed by a perfectly black body having radius 'R' as shown in figure is $= 4\pi R^2 S \longrightarrow (2)$

In the absence of atmosphere,

$$4\pi R^2 S = 4\pi r^2 E \times 60 \longrightarrow (3)$$

According to stefan's law

$$E = \sigma T^4 \longrightarrow (4)$$

From eq^n (3) and (4) we get

$$4\pi R^2 S = 4\pi r^2 \sigma T^4 \times 60$$

$$T^4 = \left(\frac{R}{r}\right)^2 \cdot \frac{S}{60} \cdot \frac{1}{\sigma}$$

$$\therefore T = \left[\left(\frac{R}{r}\right)^2 \frac{S}{60} \cdot \frac{1}{\sigma}\right]^{1/4} \longrightarrow (5)$$

Eq^n (5) is the expression of absolute temp^r of sun in terms of stefan's constant and solar constant.