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18. Electrostatic Field and potential

* Columb's law

Columb's law gives the value of electrostatic force between charges. If two charges q_1 and q_2 are separated

$$F \propto q_1 q_2$$

or, $$F \propto \frac{1}{r^2}$$

Combining both

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Here ε_0 is a constant called permittivity of free space.

If the charges were placed in any medium then the expression for force will be

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

Here ε is called permittivity of the medium

In vector form,

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

or, $$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\frac{\vec{r}}{r} \quad \left[\because \hat{r} = \frac{\vec{r}}{|\vec{r}|}\right]$$

$$\therefore \vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}$$

Note: $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2/C^2$

$$\varepsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$$

* Electric field intensity (E)

Electric field intensity at a point is the electrostatic force per unit charge experienced by an infinitesimally small positive test charge placed at that point.

i.e. $E = \lim_{q\to0}\frac{F}{q}$.

Here, the test charge is taken very small in order that it does not disturb the given charge distribution. The mathematical expression for magnitude of electric field intensity at any point is given by.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}$$

In vector form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$$

* Volume charge density (ρ)

Volume charge density is the charge stored in unit of volume.

i.e. $\rho = \frac{dq}{dv}$

$dq = \rho\, dv$

$\therefore q = \int \rho\, dv$

* Surface charge density (σ)

Surface charge density is the charge per unit surface area.

i.e. $\sigma = \frac{dq}{ds}$

$dq = \sigma\, ds$

$q = \int \sigma\, ds$

* Linear charge density (λ)

Linear charge density is the charge per unit length.

i.e. $\lambda = \frac{dq}{dl}$

$dq = \lambda\, dl$

$q = \int \lambda\, dl$

* Electric flux (ϕ)

Electric flux is the no. of electric lines of force crossing the given surface normally.

Mathematically, for small surface elements

$$d\phi_E = \vec{E}\, d\vec{s}$$

$$\therefore \phi_E = \int \vec{E}\, d\vec{s}$$

* Gauss law in electrostatics:

Gauss law in electrostatics states that the electric flux crossing a closed surface is equal to $\frac{1}{\varepsilon_0}$ total charge enclosed by the surface.

i.e. $\phi_E = \frac{1}{\varepsilon_0} q$

$$\therefore \int \vec{E}\, d\vec{s} = \frac{1}{\varepsilon_0} q$$

* Gauss law in differential form:

Gauss law in integral form is

$$\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} q$$

or, $\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \int \rho\, dv \quad [\because q = \int \rho\, dv]$

Using Gauss divergence theorem,

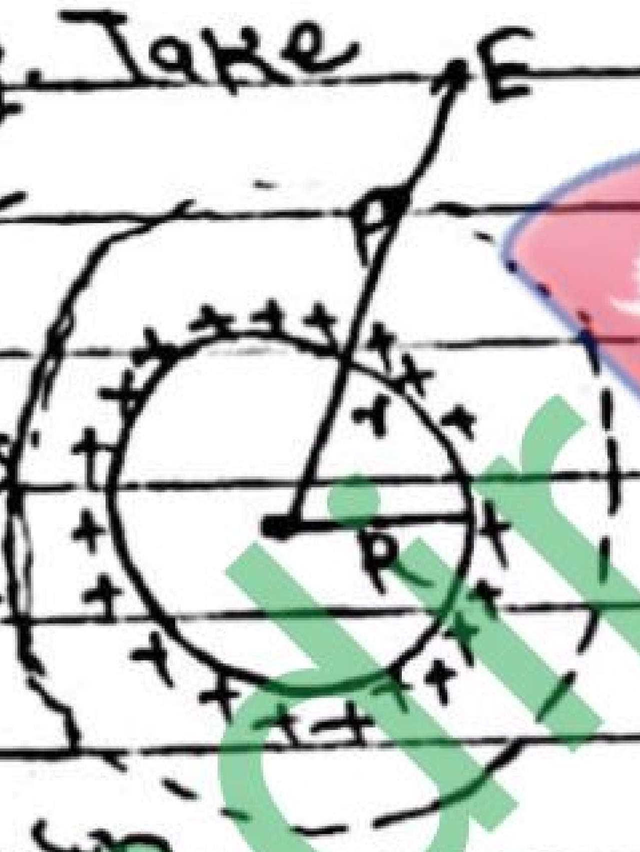
$$\int (\nabla \cdot \vec{E})\, dv = \frac{1}{\varepsilon_0} \int \rho\, dv \quad \left[\because \int_s \vec{F} \cdot d\vec{s} = \int_v (\nabla \cdot \vec{F})\, dv\right]$$

$$\therefore \int \left(\nabla \cdot \vec{E} - \frac{1}{\varepsilon_0} \rho\right) dv = 0$$

* Application of Gauss law

(1) Electric field product by spherical charge conductor

Let us consider a charged sphere of radius R and containing charge q. Take a point p at the distance r from the centre of sphere. In order to use Gauss law, let us draw an imaginary spherical Gaussian surface of radius r as shown in figure.



From Gauss law

$$\phi_E = \frac{1}{\epsilon_0} q$$

or, $\oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} q$

or, $\oint E\, ds = \frac{1}{\epsilon_0} q$

or, $E \oint ds = \frac{1}{\epsilon_0} q$

or, $E\, 4\pi r^2 = \frac{1}{\epsilon_0} q$

or, $E = \frac{1}{\epsilon_0} \frac{q}{4\pi r^2}$

$$\boxed{\therefore E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}}$$

This is the required expression for electric field at point outside the surface sphere.

Condition-1

⇒ If the point p lies on the surface of sphere the charged surface itself acts as the Gaussian surface. So $r = R$, then

$$\boxed{E_S = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}}$$

Condition-2

⇒ If the point p lies inside the charge sphere then the Gaussian surface will enclose no charge, then

$$\phi_E = \frac{1}{\varepsilon_0} \times 0$$

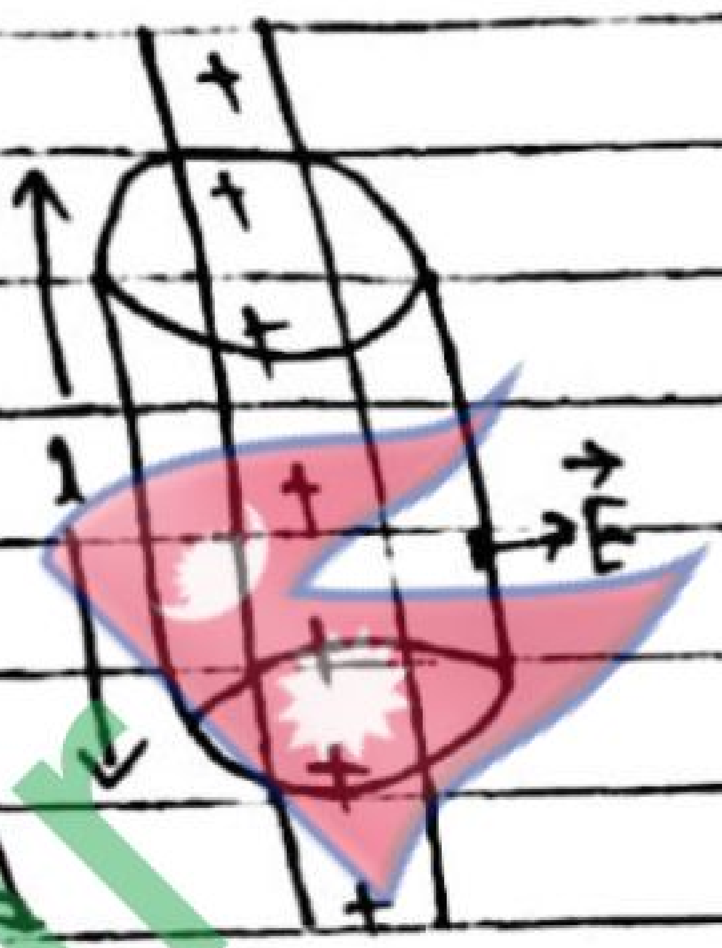
$$\int E_{in} \, d\vec{s} = 0$$

$$\therefore E_{in} = 0$$

It shows that there is no electric field inside the charge sphere.

*2. Electric field produced by line charge (For linearly charged body)

Let us consider a linearly charged body having linear charge density λ. In order to find the electric field at distance r from the charged body draws an imaginary cylindrical Gaussian surface of radius r with the charged body along the axis shown in fig.



From Gauss law,

$\phi_E = \frac{1}{\varepsilon_0} q$, here q is charge enclosed by Gaussian surface

or, $\int \vec{E}\, d\vec{s} = \frac{1}{\varepsilon_0} q$

or, $E \int ds = \frac{1}{\varepsilon_0} q$

or, $E . 2\pi r l = \frac{1}{\varepsilon_0} q$

or, $E = \frac{1}{\varepsilon_0} \frac{q}{2\pi r l}$

or, $E = \frac{1}{\varepsilon_0} \frac{\lambda}{2\pi r}$ $[\because \lambda = q/l)$

$$\boxed{\therefore E = \frac{\lambda}{2\pi \varepsilon_0 r}}$$

$\boxed{1 \text{ coulomb} = 10^6 C}$

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(26)

q→ An infinitely long wire is stretch horizontally 4m above the surface of the earth. It has a charge of 1 micro-coulomb per cm of its length. Calculate the electric field at a point on earth vertically below the earth.

[T.U 2061]

⇒ Solution,

Here,

Charge per unit length $\lambda = \frac{10^{-6}}{10^{-2}} = 10^{-4}$ C/m

$r = 4m$

Electric field, $E = \frac{\rho}{2\pi\epsilon_0 r}$

$= \frac{10^{-4}}{2\pi \times 8.85 \times 10^{-12} \times 4}$

$= 4.5 \times 10^{5}$ N/C Ans

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3 Electric field due to plane charged surface

→ Consider a plane charged surface with surface charge density σ. In order to find the electric field at a point near the surface consider an imaginary cylindrical Gaussian surface as shown in figure. If q be the charge enclosed by Gaussian surface then from Gauss law

$$\oint_s \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}q$$

$$E\int ds = \frac{1}{\varepsilon_0}q$$

$$E\cdot A = \frac{1}{\varepsilon_0}q \quad [\because \text{where A is area of crosssection}]$$

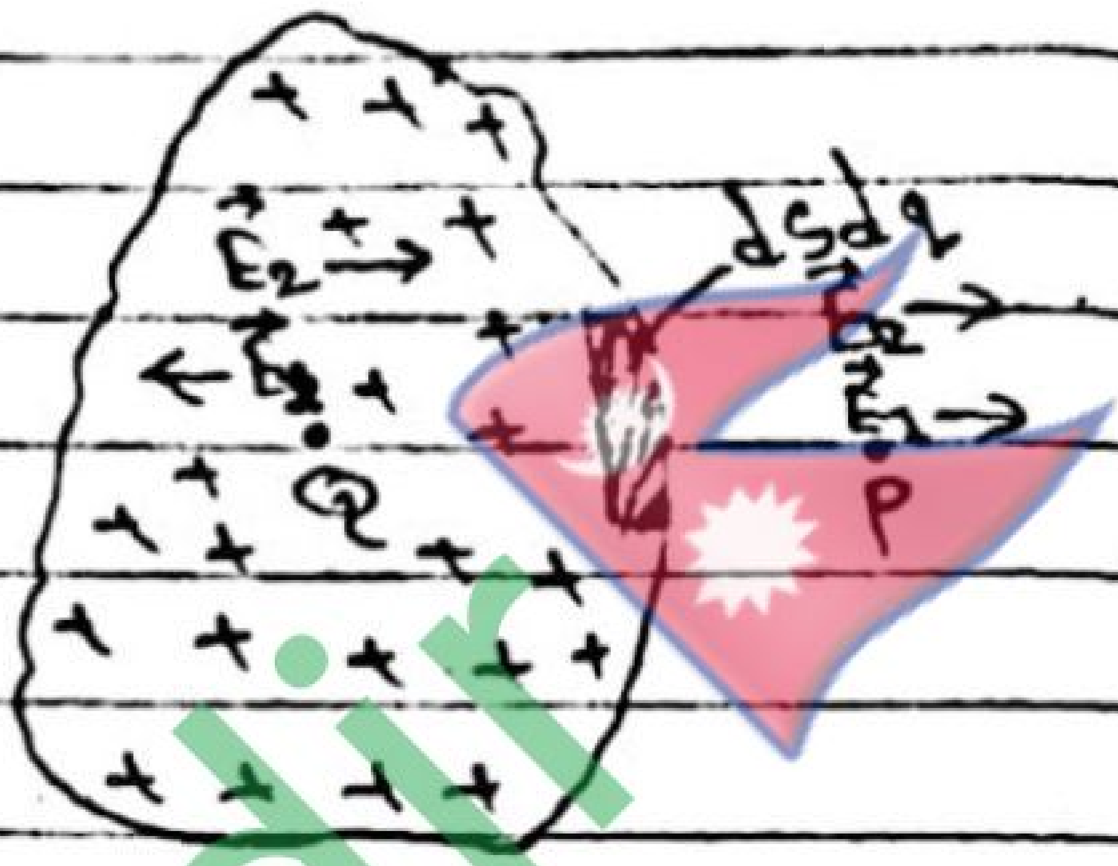
$$E = \frac{1}{\varepsilon_0}\frac{q}{A}$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \left[\because \sigma = \frac{q}{A}\right]$$

* Mechanical pressure on the surface of charged conductor

Consider a plane charged surface with surface charge density σ. Take a small region on the surface with area 'ds' and containing charge dq. If

E_1 and E_2 be the electric fields produced by charge on 'ds' and rest charge on surface then for a point outside of the surface



$E = E_1 + E_2$

$\frac{\sigma}{\varepsilon_0} = E_1 + E_2 \longrightarrow (I)$

For a point just inside of the surface net electric field '6', zero so

$E_1 - E_2 = 0$

$E_1 = E_2$

Now from (I)

$\frac{\sigma}{\varepsilon_0} = 2E_2$

$E = \frac{\sigma}{2\varepsilon_0}$

Now, the force on small region 'ds' will be

$dF = dq\, E_2$

or, $dF = dq \frac{\sigma}{2\varepsilon_0}$

Now, pressure becomes

$p = \frac{dF}{ds}$

$= \frac{dq \cdot \sigma/2\varepsilon_0}{ds}$

$$= \frac{\sigma \cdot \sigma}{2\varepsilon_0} \quad \left[\because \sigma = \frac{dq}{ds}\right]$$

$$\boxed{\therefore P = \frac{\sigma^2}{2\varepsilon_0}}$$

In terms of net electric field E

$$P = \frac{1}{2}\left(\frac{\sigma}{\varepsilon_0}\right)^2 \times \varepsilon_0 = \frac{1}{2}E^2\varepsilon_0 \quad \left[\because \frac{\sigma}{\varepsilon_0} = E\right]$$

$$\boxed{\therefore P = \frac{1}{2}\varepsilon_0 E^2}$$

* Poisson's and laplace eq^n

Poisson's equation is the relation between electric potential (ϕ) and volume charge density (ρ). The poisson's equation is

$$\nabla^2\phi = \frac{-\rho}{\varepsilon_0}$$

Proof: If $\vec{E}$ be the electric field then from Gauss law in differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \longrightarrow (i)$$

Also, electric field and potential are related by

$$\vec{E} = -\nabla\phi$$

So eq^n (i) becomes

$$\nabla \cdot (-\nabla\phi) = \frac{\rho}{\varepsilon_0}$$

$$-\nabla^2\phi = \frac{\rho}{\varepsilon_0}$$

or, $$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

This is the required poission's equation. For chargefree region $\rho = 0$. so above eqn. becomes

$$\nabla^2\phi = 0$$

This is the required Laplace's equation.

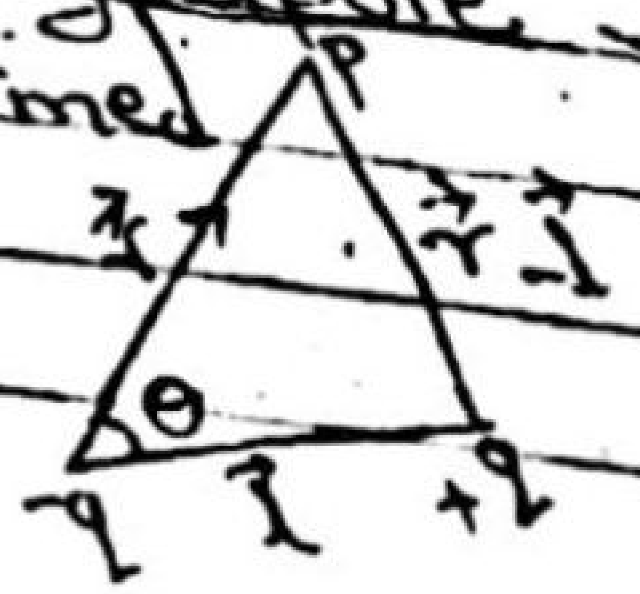
* Electric dipole :- Electric dipole is a configuration formed by separating two equal and opposite charges by small distance.

Fig: Electric dipole

* Dipole moment (p) :- Dipole moment is the product of magnitude of either charge in dipole and the distance between charges i.e. $p = ql$

It unit is column-meter (C-m).

* Electric potential produced by dipole :- Consider an electric dipole formed by two charges of magnitude 'q' having separation l. Take



a point P at separation $\vec{r}$ from '$-q$' as shown in figure.
Here, net electric potential at point P will be

$$V = V_0 + V_-$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}-\vec{l}|} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\vec{r}-\vec{l}} - \frac{q}{r}\right) \longrightarrow (i)$$

Here,

$$\frac{1}{|\vec{r}-\vec{l}|} = \frac{1}{\sqrt{r^2 - 2rl\cos\theta + l^2}}$$

$$= (r^2 - 2rl\cos\theta + l^2)^{-1/2}$$

$$= \left[r^2\left(1 - \frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right)\right]^{-1/2}$$

$$= r^{-1}\left[1 - \frac{2l\cos\theta}{r}\right]^{-1/2}$$

[$\because$ Neglecting small term $\frac{l^2}{r^2}$]

$$= r^{-1}\left[1 - \left(-\frac{1}{2}\right)\frac{2l\cos\theta}{r}\right]$$

[using binomial expansion]

$$= \frac{1}{r}\left[1 + \frac{l\cos\theta}{r}\right]$$

$$= \frac{1}{r} + \frac{l\cos\theta}{r^2}$$

Eqn (i) becomes,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{l\cos\theta}{r^2} - \frac{1}{r}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{l\cos\theta}{r^2}$$

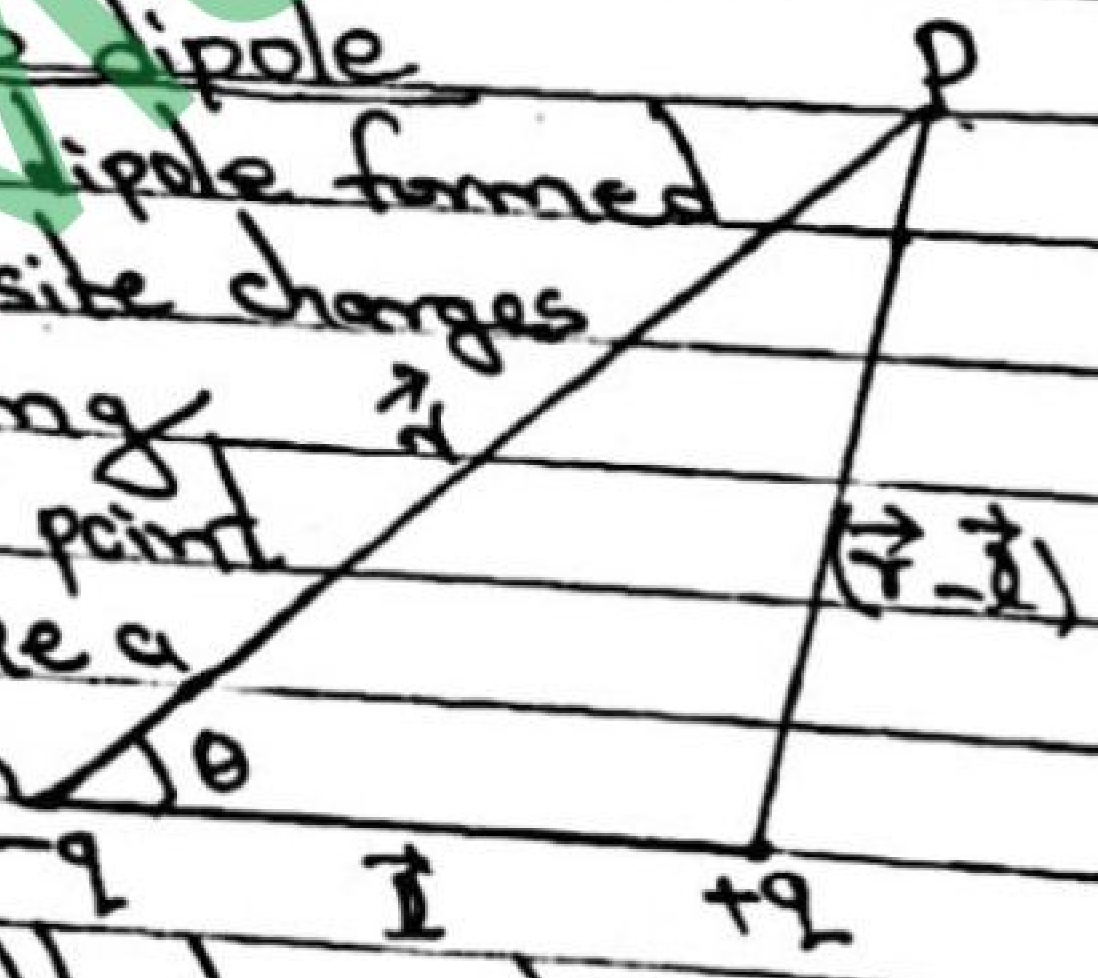
$$\therefore \boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}} \quad [\because p = ql]$$

This is the required expression potential due to electric dipole.

\# Note: In case of single charge i.e. monopole $V \propto \frac{1}{r}$ and in case of dipole $V \propto \frac{1}{r^2}$

* Electric field due to dipole

Consider an electric dipole formed by two equal and opposite charges of magnitude q having separation $\vec{l}$. Take a point P at separation $\vec{l}$. Take a point P P at separation $\vec{r}$ as shown in fig.



Here, total electric field at point P is given by

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

or, $$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{(\vec{r} - \vec{l})}{|\vec{r} - \vec{l}|^3} - \frac{q}{4\pi\varepsilon_0}\cdot\frac{\vec{r}}{r^3}$$

$$\therefore \vec{E} = \frac{q}{4\pi\epsilon_0}\left[\left(\frac{\vec{r}-\vec{l}}{|\vec{r}-\vec{l}|^3}\right) - \frac{\vec{r}}{r^3}\right]$$

$$\therefore \vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{\vec{r}-\vec{l}}{|\vec{r}-\vec{l}|^3} - \frac{\vec{r}}{r^3}\right] \longrightarrow (1)$$

Here,

$$\frac{1}{|\vec{r}-\vec{l}|^3} = \frac{1}{\left(\sqrt{r^2 - 2rl\cos\theta + l^2}\right)^3}$$

$$= \frac{1}{(r^2 - 2rl\cos\theta + l^2)^{3/2}}$$

$$= \frac{1}{\left[r^2\left(1 - \frac{2l\cos\theta}{r} + \frac{l^2}{r^2}\right)\right]^{3/2}}$$

$$= \frac{1}{r^{2\times\frac{3}{2}}\left(1 - \frac{2l\cos\theta}{r}\right)^{3/2}}$$

$\left[\because \text{Neglecting smaller terms } \frac{l^2}{r^2}\right]$

$$= \frac{1}{r^3}\left(1 - \frac{2l\cos\theta}{r}\right)^{-3/2}$$

$$= \frac{1}{r^3}\left[1 - \left(\frac{-3}{2}\right)\frac{2l\cos\theta}{r}\right]$$ [Using binomial expansion & neglecting smaller terms]

$$= \frac{1}{r^3} + \frac{3l\cos\theta}{r^4}$$

Now eq^n (1) becomes,

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$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[(\vec{r}-\vec{l})\left(\frac{1}{r^3}+\frac{3l\cos\theta}{r^4}\right)-\frac{\vec{r}}{r^3}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\vec{r}}{r^3}+\frac{3l\cos\theta\,\vec{r}}{r^4}-\frac{\vec{l}}{r^3}-\frac{3l\cos\theta\,\vec{l}}{r^4}-\frac{\vec{r}}{r^3}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{3l\cos\theta\,\vec{r}}{r^4}-\frac{\vec{l}}{r^3}\right]$$

[Neglecting the term containing l^2]

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{3q\vec{l}\cos\theta\,\vec{r}}{r^4}-\frac{q\vec{l}}{r^3}\right]$$

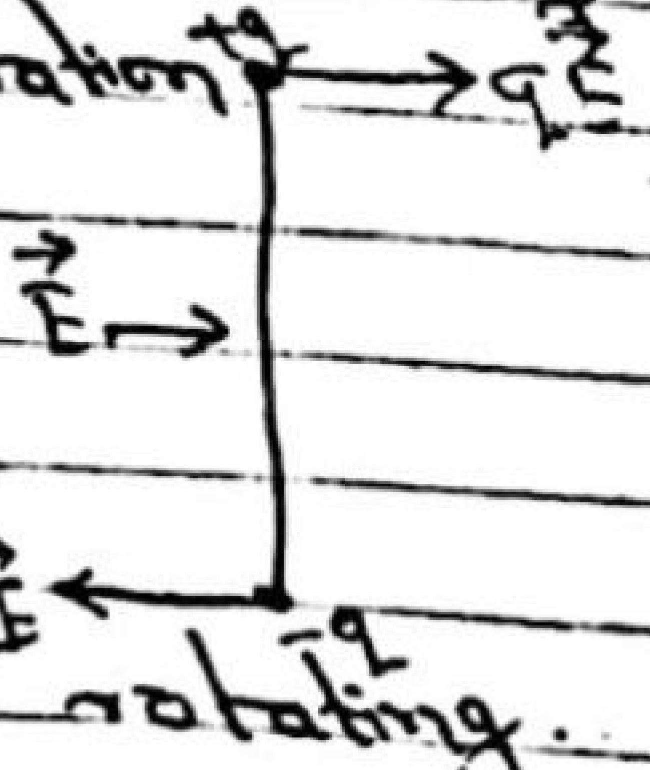
$$\therefore \vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{3\vec{P}\cos\theta\,\vec{r}}{r^4}-\frac{\vec{P}}{r^3}\right]$$

[$\because \vec{P} = q\vec{l}$ is dipole moment]

Which is the required expression for electric field ~~potential~~ produced by a dipole.

* Torque Experienced by a dipole in electric field

Consider an electric dipole formed by two charges +q and -q with separation $\vec{l}$ between them. When an external electric field $\vec{E}$ is applied these two charges experience equal but opposite forces. So, the dipole starts rotating.



Taking reference from figure

$$\vec{\tau} = -\vec{l} \times \vec{F}$$

$\vec{\tau} = \vec{l} \times q\vec{E}$

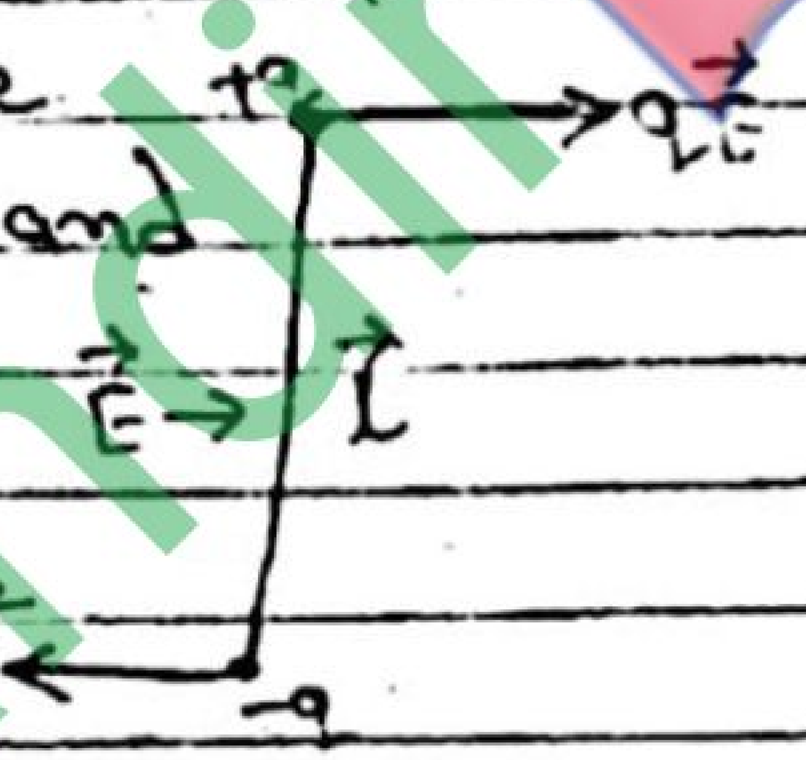
$\vec{\tau} = \vec{p} \times \vec{E} \quad [\because \vec{p} = q\vec{l}]$

So, magnitude of torque becomes

$\tau = pE\sin\theta$

* **Potential Energy of a dipole**

Consider an electric dipole formed by two charges +q and -q having separation $\vec{l}$. When an external electric field $\vec{E}$ is applied, the dipole experiences a torque given by



$\tau = pE\sin\theta$ (Magnitude)

Now, small amount of workdone for small angle of rotation $d\theta$ will be

$dW = \tau . d\theta$

For angle of rotation from θ_1 to θ_2

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta$$

$$= \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta$$

$$= pE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$\therefore W = -pE(\cos\theta_2 - \cos\theta_1)$$

This equation gives the workdone for rotation of dipole. for simplicity take $\theta_1 = 90°$, $\theta_2 = \theta°$

$W = -PE\cos\theta$

This work stores in the form of potential energy (U), so

$U = -PE\cos\theta$

$$\boxed{\therefore U = -\vec{P}\cdot\vec{E}}$$

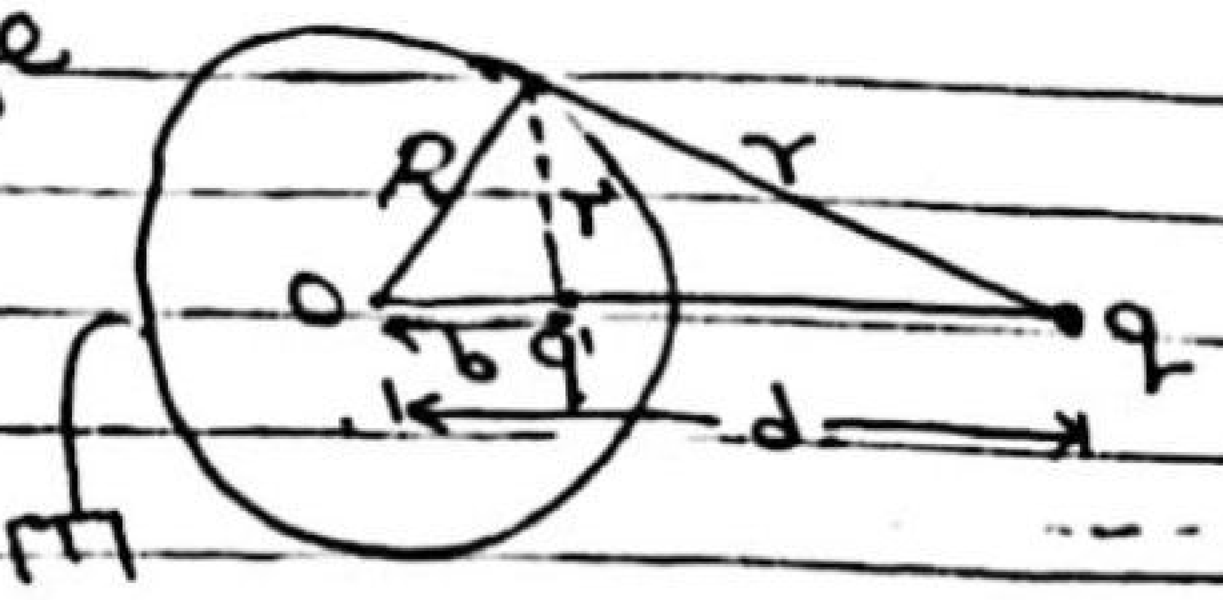
(J)

Electrical image

Electrical image is the charge ~~indicud~~ induced when a charge or charge distribution is placed near a conductor. The image remain there until original charge stays there. In order to calculate the electric potential and field we have to consider both original charge and its electrical image.

* Value and position of image charge for conducting sphere

Suppose a charge q is located at distance 'd' from the centre of a conducting



sphere of radius 'R'. Let q' be the its electrical image located at distance b from the centre as shown in fig.

Since electric potential at the surface of sphere is zero, we can write

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q'}{r'} = 0$$

or, $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{q'}{r'}\right) = 0$

or, $\frac{q}{r} + \frac{q'}{r'} = 0$

or, $\frac{q}{r} = -\frac{q'}{r'}$

or, $q^2 r'^2 = q'^2 r^2$

Using cosine formula,

or, $q^2[R^2+b^2-2Rb\cos\theta] = q'^2[R^2+d^2-2Rd\cos\theta]$

This is true for all values of θ.

Equating the coefficient of $\cos\theta$, we get

$-2Rbq^2 = -2Rdq'^2$

or, $bq^2 = dq'^2$

or, $\frac{b}{d} = \frac{q'^2}{q^2} = \alpha$ (say)

So, $b = \alpha d$ and $q'^2 = \alpha q^2 \longrightarrow (1)$

Now, Equating the remaining parts of above equation, we get

$q^2(R^2+b^2) = q'^2(R^2+d^2)$

or, $q^2(R^2+\alpha^2 d^2) = \alpha q^2(R^2+d^2)$
[using $b=\alpha d$ and $q'^2 = \alpha q^2$]

or, $\alpha^2 d^2 + R^2 = \alpha(R^2+d^2)$

or, $\alpha^2 d^2 - \alpha(R^2+d^2) + R^2 = 0$
Which is quadratic in α, its solution will be,

So, $\alpha = \dfrac{-[-(R^2+d^2)] \pm \sqrt{(R^2+d^2)^2 - 4d^2R^2}}{2d^2}$

or, $\alpha = \dfrac{(R^2+d^2) \pm \sqrt{(R^2-d^2)^2}}{2d^2}$

or, $\alpha = \dfrac{(R^2+d^2) \pm (R^2-d^2)}{2d^2}$

Taking +ve sign,

or, $\alpha = \dfrac{R^2+d^2+R^2-d^2}{2d^2}$

$= \dfrac{R^2}{d^2}$

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Taking -ve sign,

$$\alpha = \frac{R^2+d^2-R^2+d^2}{2d^2}$$

$= 1$ (Not valid)

Now eq^n (1) becomes,

$$b = \alpha d = \frac{R^2}{d^2} \cdot d = \frac{R^2}{d}$$

and $q'^2 = \alpha q^2 = \frac{R^2}{d^2} q^2$

$$q' = \pm \frac{R}{d} q$$

$\therefore q' = -\frac{R}{d} q$ [Positive sign is not valid]

Hence, value of image charge

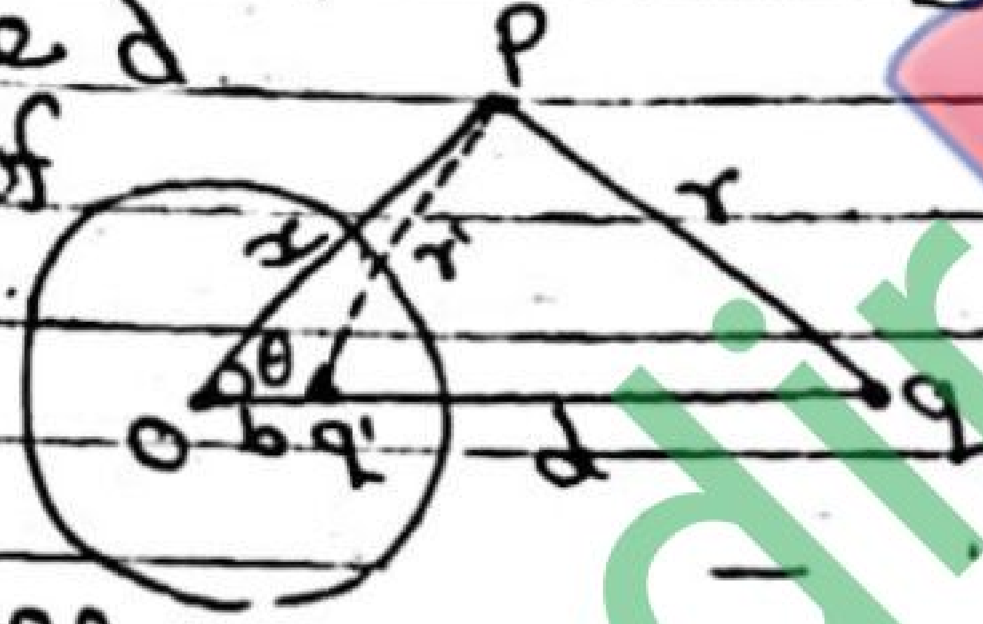
$$q' = -\frac{R}{d} q$$

Position of image charge (b)

$$= \frac{R^2}{d}$$

Electric potential and field due to a point charge near conducting sphere

Let us consider a point charge q located at distance d from the centre of a conducting sphere of radius R. In this case, an electrical image is induced within the sphere for which



$$q' = -\frac{R}{d}q,$$

$$b = \frac{R^2}{d}$$

For a point P at distance x from the centre of sphere. The electric potential will be

V = potential due to q + potential due to q'

or, $$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{q'}{r'}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{(-R/d)q}{r'}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{R}{d}\cdot\frac{1}{r'}\right]$$

$$\therefore V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2+d^2-2xd\cos\theta}} - \frac{R}{d}\frac{1}{\sqrt{x^2+b^2-2xb\cos\theta}}\right]$$

Which gives the electric potential at point p.

Now, the component of electric field along op will be

$$E_x = -\frac{\partial V}{\partial x}$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[\left(-\frac{1}{2}\right)(x^2+d^2-2xd\cos\theta)^{-3/2}(2x-2d\cos\theta) - \frac{R}{d}\left(-\frac{1}{2}\right)(x^2+b^2-2xb\cos\theta)^{-3/2}(2x-2b\cos\theta)\right]$$

$$\therefore E_x = \frac{q}{4\pi\varepsilon_0}\left[\frac{(x-d\cos\theta)}{(x^2+d^2-2xd\cos\theta)^{3/2}} + \frac{R}{d}\frac{(x-b\cos\theta)}{(x^2+b^2-2xb\cos\theta)^{3/2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{x-d\cos\theta}{(x^2+d^2-2xd\cos\theta)^{3/2}} - \frac{R}{d}\frac{x-b\cos\theta}{(x^2+b^2-2xb\cos\theta)^{3/2}}\right]$$

Now, the component of electric field perp. to E_x will be

$$E_\theta = -\frac{1}{x}\frac{\partial V}{\partial \theta}$$

$$= \frac{-q}{4\pi\varepsilon_0 x}\left[\left(-\frac{1}{2}\right)(x^2+d^2-2xd\cos\theta)^{-3/2}(2xd\sin\theta) - \frac{R}{d}\left(-\frac{1}{2}\right)(x^2+b^2-2xb\cos\theta)^{-3/2} 2xb\sin\theta\right]$$

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$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{-d\sin\theta}{(x^2+d^2-2xd\cos\theta)^{3/2}} + \frac{R}{d}\frac{b\sin\theta}{x^2+b^2-2xb\cos\theta)^{3/2}}\right]$$

$$\therefore E_\theta = \frac{q}{4\pi\varepsilon_0}\left[\frac{d\sin\theta}{x^2+d^2-2xd\cos\theta)^{3/2}} - \frac{R}{d}\frac{b\sin\theta}{x^2+b^2-2xb\cos\theta)^{3/2}}\right]$$

Now the net electric field produced will be

$$\vec{E} = E_x\hat{x} + E_\theta\hat{\theta}$$

Its magnitude is

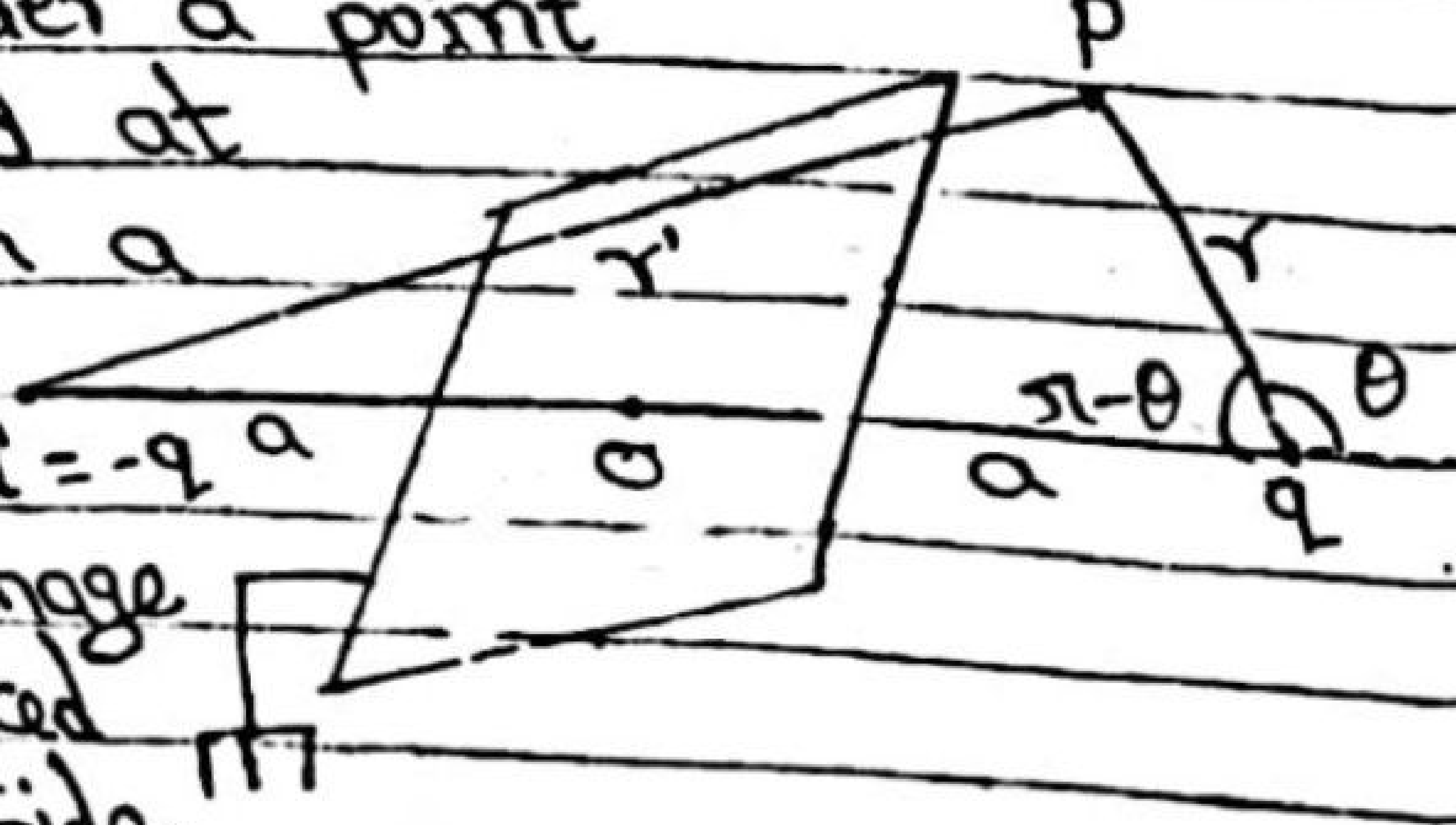
$$E = \sqrt{E_x^2 + E_\theta^2}$$

And direction will be

$$\alpha = \tan^{-1}\left(\frac{E_\theta}{E_x}\right)$$

Electric potential and field due to a charge near plane conductor

Let us consider a point charge q placed at distance 'a' from a plane conductor. In this case, $q' = -q$ an electrical image $q'(=-q)$ is produced on the opposite side of plane conductor as shown in figure.



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Now electric potential at point p is given by

V = potential due to q + ~~da~~ potential due to q'.

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{r'}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2+(2a)^2-2r\cdot 2a\cos(\pi-\theta)}}\right]$$

$$\therefore V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2+4a^2+4ra\cos\theta}}\right]$$

This is the required expression for potential produced at point p.

Now, the component of electric field along r is given by

$$E_r = -\frac{\partial V}{\partial r}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r^2} + \frac{1}{2}(r^2+4a^2+4ra\cos\theta)^{-3/2}(2r+4a\cos\theta)\right]$$

$$E_r = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{r+2a\cos\theta}{(r^2+4a^2+4ra\cos\theta)^{3/2}}\right]$$

And, the component of electric field perp. to E_r will be

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

$$= -\frac{q}{4\pi\varepsilon_0 r}\left[0-\left(-\frac{1}{2}\right)(r^2+4a^2+4ra\cos\theta)^{-3/2}(4ra(-\sin\theta))\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\sin\theta}{(r^2+4a^2+4ra\cos\theta)^{3/2}}\right]$$

$$\therefore E_\theta = \frac{q}{2\pi\varepsilon_0}\left[\frac{a\sin\theta}{(r^2+4a^2+4ra\cos\theta)^{3/2}}\right]$$

Then, the net electric field produced becomes,

$$\vec{E} = E_r\hat{r} + E_\theta\hat{\theta}$$

Its magnitude is

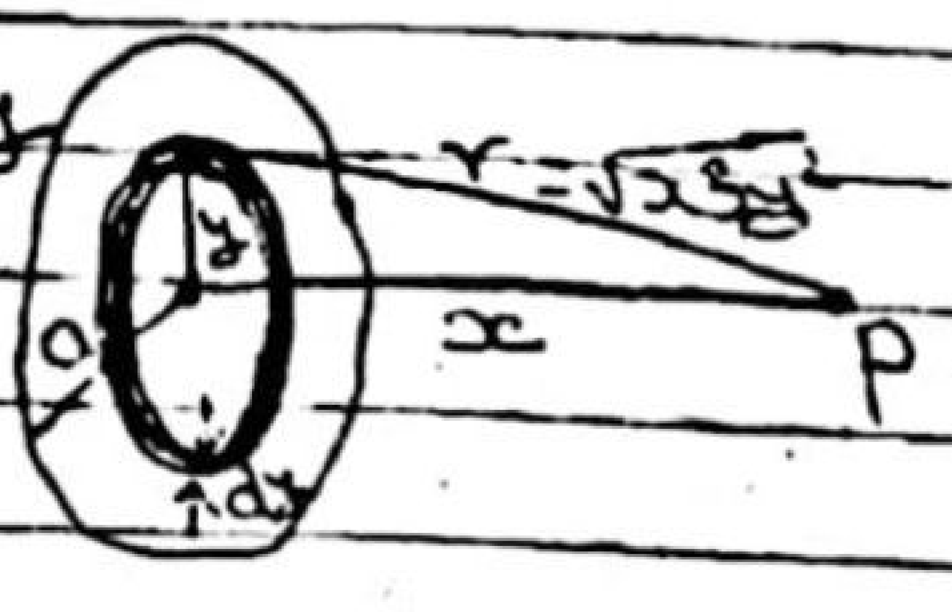
$$E = \sqrt{E_r^2 + E_\theta^2}$$

And direction is

$$\alpha = \tan^{-1}\left(\frac{E_\theta}{E_r}\right)$$

④ # Electric potential produced by uniformly charge disc

Consider a uniformly charge disc having surface charge density σ and radius a. Take a point 'p' on the axis of disc at distance x as shown in figure. Draw an elementary ring of radius y



and thickness 'dy' on the disc.
Now, the small electric potential at point P produced by the small elementary ring will be

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$$

$$\text{or, } dV = \frac{1}{4\pi\varepsilon_0} \frac{\sigma dA}{\sqrt{x^2+y^2}} \quad \left(\sigma = \frac{dq}{dA}\right)$$

$$\text{or, } dV = \frac{1}{4\pi\varepsilon_0} \frac{\sigma 2\pi y dy}{\sqrt{x^2+y^2}} \quad [\because dA = 2\pi y dy]$$

Now, total electric potential at point P will be

$$V = \int_0^a \frac{\sigma}{4\varepsilon_0} \frac{2y dy}{\sqrt{x^2+y^2}}$$

$$\text{or, } V = \frac{\sigma}{4\varepsilon_0} \int_0^a \frac{2y dy}{\sqrt{x^2+y^2}} \longrightarrow (1)$$

Here for integral part,
suppose $x^2+y^2 = z$
$2y dy = dz$
Now, eq^n. (1) becomes,

$$V = \frac{\sigma}{4\varepsilon_0} \int_{x^2}^{x^2+a^2} \frac{dz}{\sqrt{z}}$$

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$$= \frac{\sigma}{4\varepsilon_0} \int_{x^2}^{x^2+a^2} z^{-1/2} dz$$

$$= \frac{\sigma}{4\varepsilon_0} \left[\frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_{x^2}^{x^2+a^2}$$

$$= \frac{\sigma}{4\varepsilon_0} \left[\frac{z^{\frac{1}{2}}}{\frac{1}{2}} \right]_{x^2}^{x^2+a^2}$$

$$= \frac{\sigma}{4\varepsilon_0} \cdot 2 \left[\sqrt{x^2+a^2} - \sqrt{x^2} \right]$$

$$V = \frac{\sigma}{2\varepsilon_0} \left(\sqrt{x^2+a^2} - x \right)$$

Which is the required expression for the electric potential produced at point P.

If point P lies at the centre of disc $x = 0$, so

$$V_{centre} = \frac{\sigma a}{2\varepsilon_0}$$

For electric field (E)
We have,

$$E = -\frac{\partial V}{\partial x}$$

or, $E = -\frac{\sigma}{2\epsilon_0}\left[\frac{1}{2}(x^2+a^2)^{1/2-1} \cdot 2x - 1\right]$

or, $E = -\frac{\sigma}{2\epsilon_0}\left[x(x^2+a^2)^{-1/2} - 1\right]$

or, $E = -\frac{\sigma}{2\epsilon_0}\left[\frac{x}{\sqrt{x^2+a^2}} - 1\right]$

$\therefore E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2+a^2}}\right]$

Electric field and potential due to linearly charged body

See page-6 : Electric field due to linearly charged body

We have,

$E = \frac{\lambda}{2\pi\epsilon_0 r}$

For electric potential

$E = -\frac{\partial V}{\partial r}$

or, $\partial V = -E\,dr$

or, $V = -\int E\,dr$

or, $V = -\int \frac{\lambda}{2\pi\epsilon_0 r}\,dr$

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or, $V = -\frac{\lambda}{2\pi\varepsilon_0}\int \frac{1}{r} dr$

For limit $r = r_A$ to r_B we get

$$V = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_A}^{r_B} \frac{1}{r} dr$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln r\right]_{r_A}^{r_B}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}(\ln r_B - \ln r_A)$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \frac{\ln r_B}{\ln r_A}$$

$$\therefore V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_A}{r_B}\right)$$

Which is the required expression for electric potential due to linearly charged body.