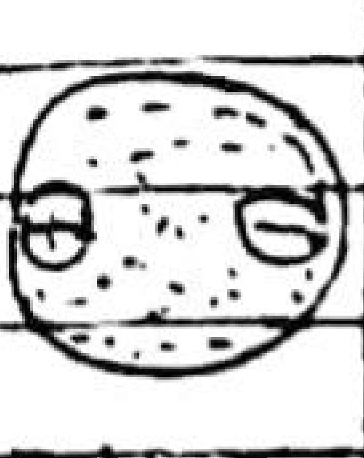
chapter 19

Electric field in dielectric (83)

\# **Dielectrics:** Dielectrics are the insulators which can be polarized by the application of external electric field. When dielectric materials are placed within the places of a capacitor, its capacitance increases.

\# Types of dielectrics

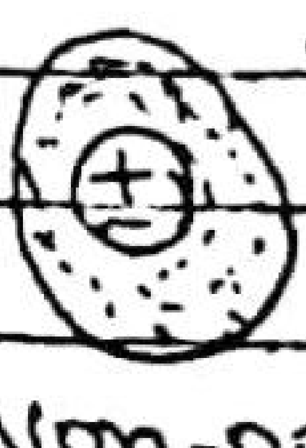
1. Polar dielectrics



polar

The dielectric with centre of +ve charge and centre of -ve charge at the different point are called polar dielectrics. Such dielectrics have permanent dipole moment. For eg: H_2O, HCl etc

2. Non-polar dielectrics



Non-polar

The dielectric with centre of +ve charge and centre of -ve charge at the same point are called non-polar dielectrics. In such dielectrics, there is no permanent dipole moment. For eg: O_2, N_2, CH_4 etc.

\# Theory of induced polarization in non-polar dielectric

In case of non-polar dielectrics, the centre of +ve charge and centre of -ve charge coincide at the same point. If an external electric field is applied, the positive charge is pushed along the field direction and negative charge is pushed opposite to field direction. In this way, the dielectric gets polarized there, the dipole moment created is found to be directly proportional to external field applied.

ie: $p \propto E$

$\therefore p = \alpha E$

Where α is a constant is called molecular polarizability.

Definition of molecular polarizability (α)

Molecular polarizability is a physical quantity that describes the extent of polarization of a molecule in responce to applied external electric field.

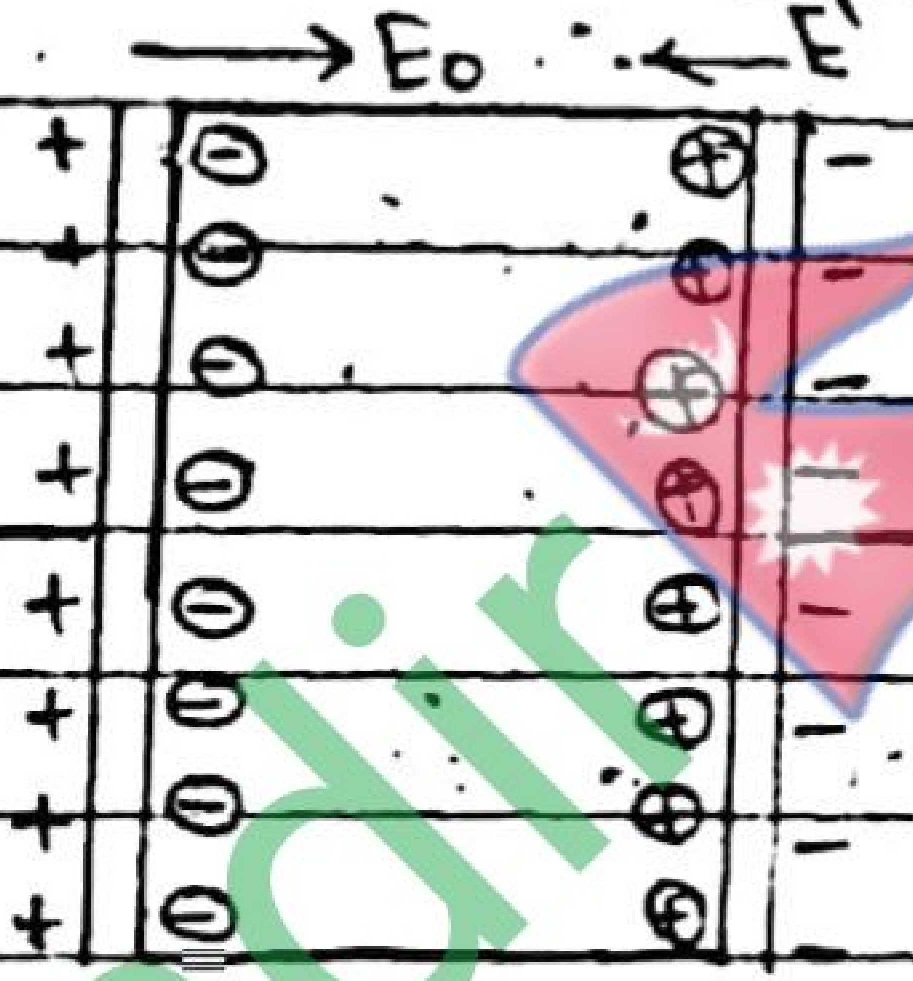
Mathematically, it is the ratio of dipole moment produced to the external electric field applied. That is

$$\alpha = \frac{p}{E}$$

The unit of α is Fm^2. Its values depend upon nature of molecule and temperature.

Dielectric placed betn. plates of charged capacitor

Let us consider E be the electric field betn. the plates of charge capacitor without dielectric. When a dielectric is placed betn. these plates, it gets polarized. Hence a new electric field E' is developed along the opposite direction. Hence net electric field between the plates of capacitor decreases. If E be the net electric field then,



$$E = E_0 - E'$$

The factor by which electric field decreases is called dielectric constant (K).

So, dielectric constant is the ratio of electric field between plates of capacitor without electric to that with dielectric

i.e. $K = \frac{E_0}{E}$

Value of dielectric constant is equal to relative permittivity of dielectric.

i.e. $K = \frac{\varepsilon}{\varepsilon_0}$.

Gauss law in dielectric

For a capacitor with charge q on each plate of area A, the Gauss Law in electrostatic takes the form.

$$\int \vec{E_0} \cdot \vec{ds} = \frac{1}{\varepsilon_0} q$$

or, $E_0 \cdot A = \frac{1}{\varepsilon_0} q$

or, $E_0 = \frac{1}{\varepsilon_0} \frac{q}{A} \longrightarrow (1)$

When a dielectric slab is placed betn the plates of capacitor a charge of opposite nature is induced there. If q' be the induced charge then,

$$\int \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0}(q-q') \longrightarrow (2)$$

or, $E \cdot A = \frac{1}{\varepsilon_0}(q-q')$

or, $E = \frac{1}{\varepsilon_0}\left(\frac{q-q'}{A}\right) \longrightarrow (3)$

By the defination of dielectric constant (K)

$$K = \frac{E_0}{E}$$

or, $K = \dfrac{\frac{1}{\varepsilon_0}\frac{q}{A}}{\frac{1}{\varepsilon_0}\left(\frac{q-q'}{A}\right)}$

or, $K = \frac{q}{q-q'}$

or, $Kq - Kq' = q$

or, $q(K-1) = Kq'$

or, $q' = \frac{q(K-1)}{K} \longrightarrow (4)$

This eq^n gives the value of induced charge. It shows that the induced charge is always less than inducing charge.

Now,

From eq^n (2) and (4), we have

$$\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0}\left(q - \frac{q(K-1)}{K}\right)$$

or, $\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0}\left[\frac{Kq - Kq + q}{K}\right]$

or, $\int \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \frac{q}{K}$

or, $\int \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0} \frac{\varepsilon_0}{\varepsilon}$

or, $\int \varepsilon \cdot \vec{E} \cdot d\vec{s} = q$

or, $\int \vec{D} \cdot d\vec{s} = \int \rho \, dv$ [Here, $\vec{D} = \varepsilon \cdot \vec{E}$]

[Here, $q = \int \rho \, dv$]

or, $\int (\nabla \cdot \vec{D}) dV = \int \rho dV$

[Using Gauss divergence theorem]

or, $\nabla \cdot D = \rho$

This is the required Gauss law in dielectrics.

(I) # Displacement vector ($\vec{D}$) or Electric Displacement

When a dielectric slab is placed inside plates of a charged capacitor, the dielectric material gets polarized. The physical quantity that measures the extent of cause of polarization of dielectric material is called displacement vector. Mathematically it is equal to surface charge density of polarizing charge

i.e. $D = \frac{q}{A}$

If $\vec{E}$ is the electric field displacement vector will be $\vec{D} = \varepsilon \vec{E}$

(I)

Polarization vector ($\vec{P}$) or electric polarization

When a electric slab is placed in polarizing field, the charges are induced there. The physical quantity that describes the extent of polarization of the molecule is called polarization vector. It is mathematically equal to surface charge density of induced charge.

i.e. $P = \frac{q'}{A}$

(II)

Relation among field vectors in dielectric ($D = \varepsilon_0 \vec{E} + \vec{P}$)

If q be the inducing charge and q' be the charge induced then Gauss law takes the form

$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0}(q - q')$$

$$\text{or, } E \cdot A = \frac{1}{\varepsilon_0}(q - q')$$

$$\text{or, } E \varepsilon_0 = \frac{q}{A} - \frac{q'}{A}$$

$$\varepsilon_0 E = D - P \quad \left[\because D = \frac{q}{A}, P = \frac{q'}{A}\right]$$

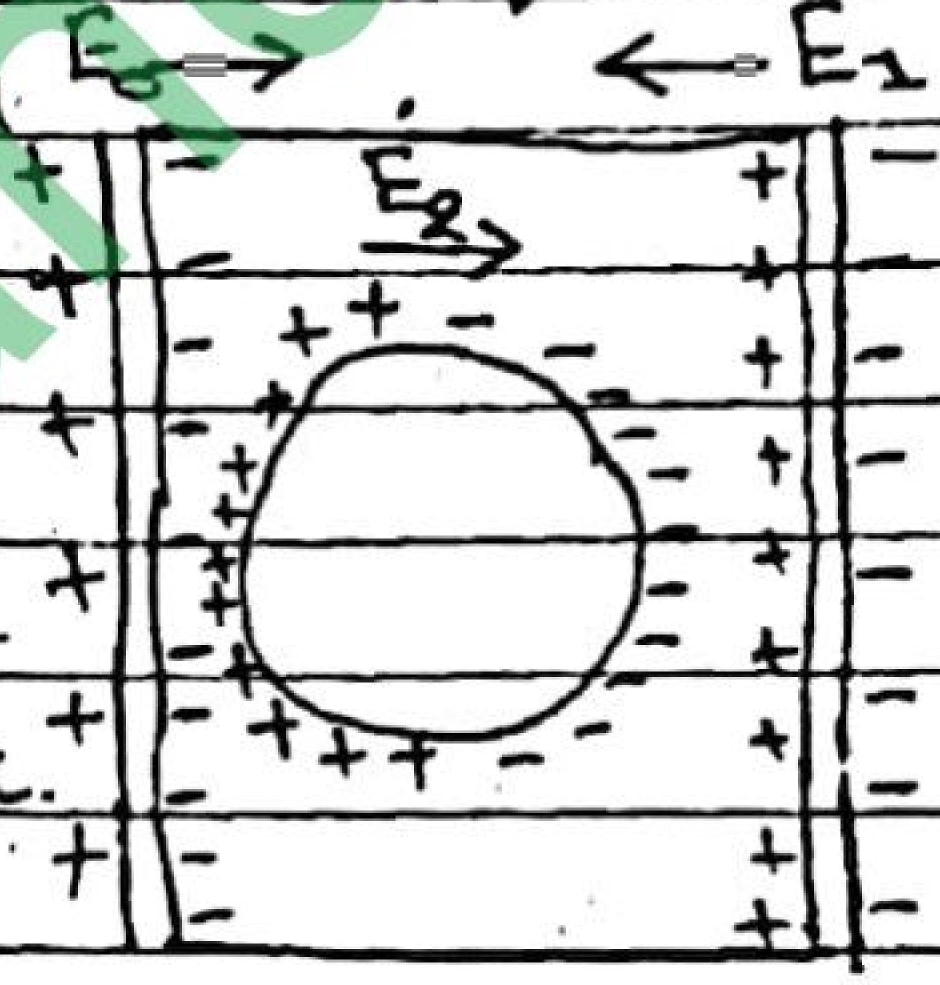
or, $D = \varepsilon_0 E + P$

This is required relation among field vectors in dielectric.
In vector form,

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

L Q

Clausius - Mossoti Relation



Clausius Mossoti relation is a mathematical relation between dielectric constant (K) and molecular polarizability (α) of the dielectric. This is given by

$$\frac{K-1}{K+2} = \frac{N\alpha}{3\varepsilon_0}.$$

Proof:-
Let us consider a dielectric material of dielectric constant 'K' and molecular polarizability 'α' placed between the plates of charge capacitor. If a dielectric has a spherical cavity as shown in figure then the net molecular field (E_m) will be

$$E_m = E_0 + E_1 + E_2 \longrightarrow (1)$$

Here,

$E_0 = \frac{\sigma}{\varepsilon_0}$ is electric field due to charge on plates

$E_1 = -\frac{P}{\varepsilon_0}$ is electric field due to charge induced charge on surface

$E_2 = \frac{P}{3\varepsilon_0}$ is electric field due to charge on surface of cavity

Now, eq^n (1) becomes

$$E_m = \frac{\sigma}{\varepsilon_0} - \frac{P}{\varepsilon_0} + \frac{P}{3\varepsilon_0}$$

or, $E_m = \frac{D - P}{\varepsilon_0} + \frac{P}{3\varepsilon_0}$ $\left[\because D = \frac{q}{A} = \sigma\right]$

or, $E_m = \frac{\varepsilon_0 E}{\varepsilon_0} + \frac{P}{3\varepsilon_0}$ $[\because D = \varepsilon_0 E + P]$

$$\therefore E_m = E + \frac{P}{3\varepsilon_0} \longrightarrow (2)$$

We have,

$D = \varepsilon_0 E + P$

or, $\varepsilon E = \varepsilon_0 E + P$ $[\because D = \varepsilon E]$

or, $K\varepsilon_0 E - \varepsilon_0 E = P$ $\left[\because K = \frac{\varepsilon}{\varepsilon_0}\right]$

or, $\varepsilon_0 E(K-1) = P$

or, $P = \varepsilon_0 E(K-1) \longrightarrow (3)$

Now, From eq^n (2) and (3), we get

$$E_m = E + \frac{\varepsilon_0 E(K-1)}{3\varepsilon_0}$$

or, $E_m = E + \frac{E(K-1)}{3}$

or, $E_m = E\left(1 + \frac{K-1}{3}\right)$

$\therefore E_m = E\left(\frac{K+2}{3}\right) \longrightarrow (4)$

If N be the no. of molecules per unit volume and p be the dipole moment of each molecule then the electric polarization P will be

$$P = N p$$

or, $P = N\alpha E_m$ $[p = \alpha E_m]$

or, $\varepsilon_0 E(K-1) = N\alpha E\left(\frac{K+2}{3}\right)$ [From (3) & (4)]

or, $\varepsilon_0 (K-1) = N\alpha\left(\frac{K+2}{3}\right)$

or, $\boxed{\frac{K-1}{K+2} = \frac{N\alpha}{3\varepsilon_0}}$

This is the required Clausius Mossotti relation.

Limitations

1. It work only for non-polar dielectric
2. It can not explain temperature dependence of molecular polarizability (α).
3. It lacks the short range interaction between molecules.

LG W/A

Langevin-Debye Relation

Clausius Mossotti relation holds true only for non-polar dielectrics. So for polar dielectrics it has to be modification by Debye. According to him, the polar dielectric has two types of polarizability :- molecular polarizability (α_m) and orientational polarizability (α_0).

So, $\alpha = \alpha_m + \alpha_0 \longrightarrow (1)$

When a dipole is placed in external electric field E the potential energy stored is given

$$U = -\vec{P}\cdot\vec{E} = -PE\cos\theta$$

According to Maxwell Boltzmann

the no. of electric dipoles within solid angle of $d\Omega$ is

$$dn = Ae^{\frac{U}{K_BT}} d\Omega$$

$$\text{or, } dn = Ae^{\frac{pE\cos\theta}{K_BT}} d\Omega$$

So total no. of dipole will be

$$n = \int_0^{\pi} Ae^{\frac{pE\cos\theta}{K_BT}} d\Omega \longrightarrow (2)$$

The component of a single dipole moment along the field direction is $p\cos\theta$. so for dn dipoles, this component will be $p\cos\theta\, dn$

Hence,

$$\Sigma p = \int_0^{\pi} p\cos\theta\, dn$$

$$\text{or, } \Sigma p = \int_0^{\pi} p\cos\theta\, Ae^{\frac{pE\cos\theta}{K_BT}} d\Omega$$

$$\therefore \Sigma p = A\int_0^{\pi} p\cos\theta \cdot e^{\frac{pE\cos\theta}{K_BT}} d\Omega$$

Now, average dipole moment will be

$$p_{av} = \frac{\Sigma p}{n}$$

$$\text{or, } p_{av} = \frac{A\int_0^{\pi} p\cos\theta\, e^{\frac{pE\cos\theta}{K_BT}} d\Omega}{A\int_0^{\pi} e^{\frac{pE\cos\theta}{K_BT}} d\Omega}$$

$$\text{or, } p_{av} = \frac{\int_0^{\pi} p\cos\theta\, e^{x\cos\theta} d\Omega}{\int_0^{\pi} e^{x\cos\theta} d\Omega}$$

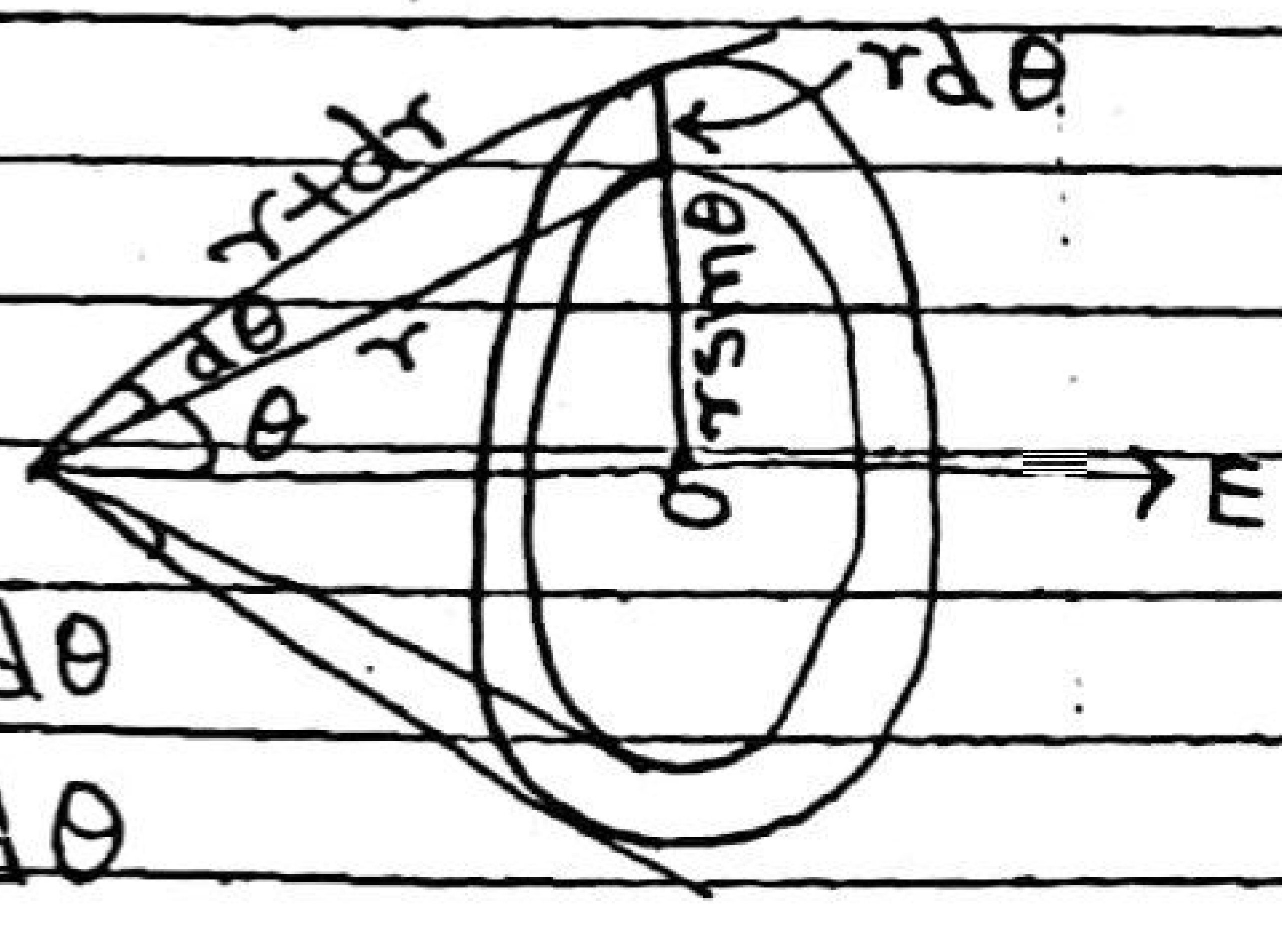
Where, $x = \frac{pE}{K_BT}$

From definition of solid angle

$$d\Omega = \frac{\text{area of elementary ring}}{r^2}$$

$$\text{or, } d\Omega = \frac{2\pi r\sin\theta\, r d\theta}{r^2}$$

$$\therefore d\Omega = 2\pi\sin\theta\, d\theta$$



Now,

$$p_{av} = \frac{\int_0^{\pi} p\cos\theta\, e^{x\cos\theta} 2\pi\sin\theta d\theta}{\int_0^{\pi} e^{x\cos\theta} 2\pi\sin\theta d\theta}$$

$$\text{or, } p_{av} = \frac{\int_0^{\pi} p\cos\theta\, e^{x\cos\theta} \sin\theta d\theta}{\int_0^{\pi} e^{x\cos\theta} \sin\theta d\theta}$$

Put $\cos\theta = a$ then

$\sin\theta d\theta = -da$

Hence, the above terms becomes,

$$p_{av} = \frac{\int_1^{-1} pae^{ax}(-da)}{\int_1^{-1} e^{ax}(-da)} = \frac{p\int_1^{-1} ae^{ax}\,da}{\int_1^{-1} e^{ax}\,da}$$

The standard value of the above integral is $\frac{x}{3}$

So, $p_{av} = p.\frac{x}{3}$

or, $p_{av} = \frac{p}{3}.\frac{pE}{K_BT} = \frac{p^2}{3K_BT}E$

Comparing this eqn. with $p = \alpha E$, we get orientational polarizability (α_0) is

$$\therefore \alpha_0 = \frac{p^2}{3K_BT}$$

Now, from eqn. (1)

$$\therefore \alpha = \alpha_{in} + \frac{p^2}{3K_BT}$$

This is required ~~eqn of~~ Langevin-Debye equation

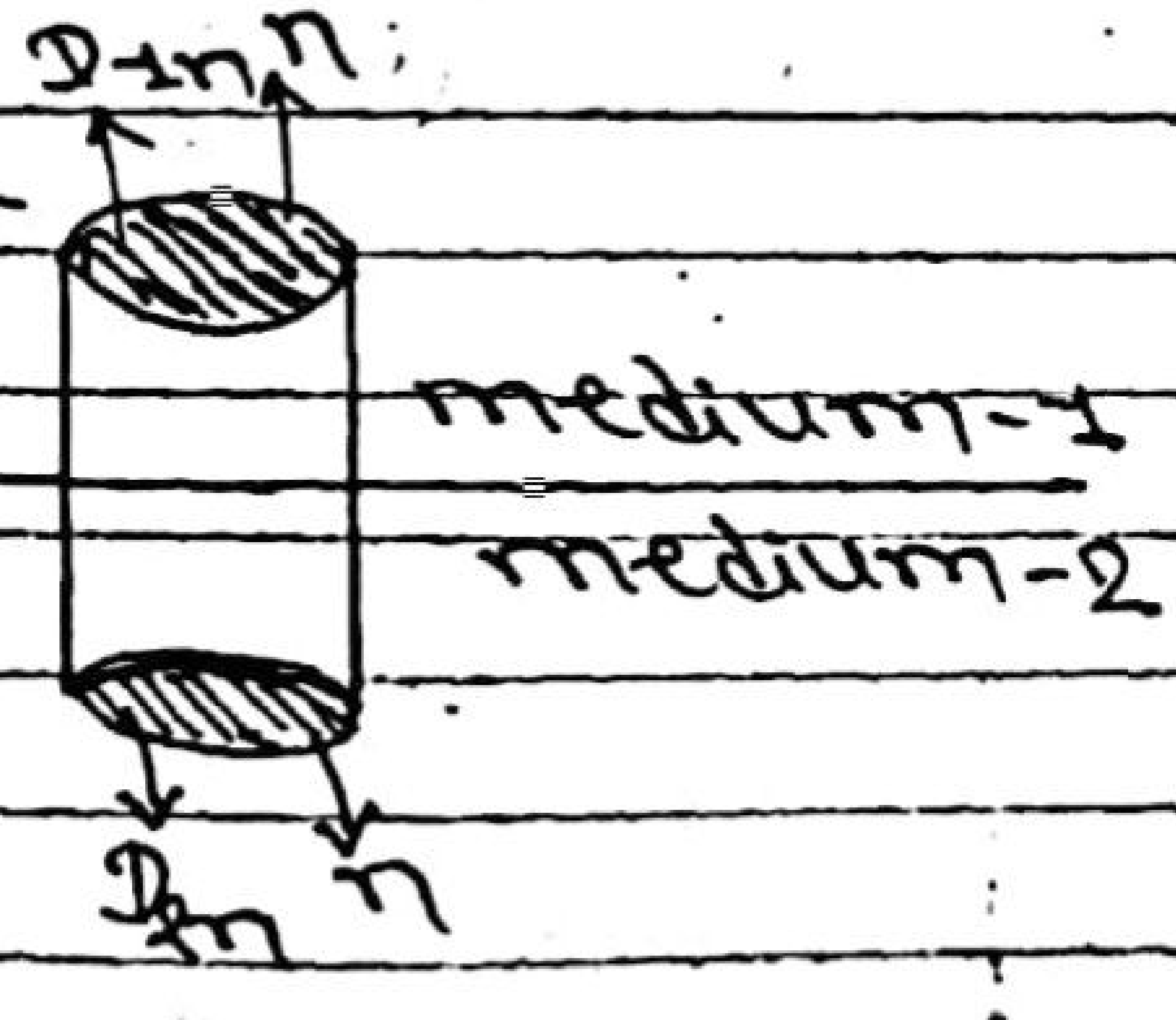
Field vectors and bounding conditions.

The electric field intensity vector (E) and electric displacement vector (D) describe the behaviour of electric field inside any medium. These two vectors are called field vectors.

When the electric field crusses in medium, there arises certain changes. The conditions obeyed by the field vectors at the boundary between two media are called boundary conditions.

Boundary condition for $\vec{D}$

:- Let us consider a cylindrical elementary region connecting medium-1 and medium-2 as shown in figure.



From Gauss law,

$$\oint \vec{D}\, \vec{ds} = 0$$

or, $\int D_{1n}\, ds = \int D_{2n}\, dS = 0$

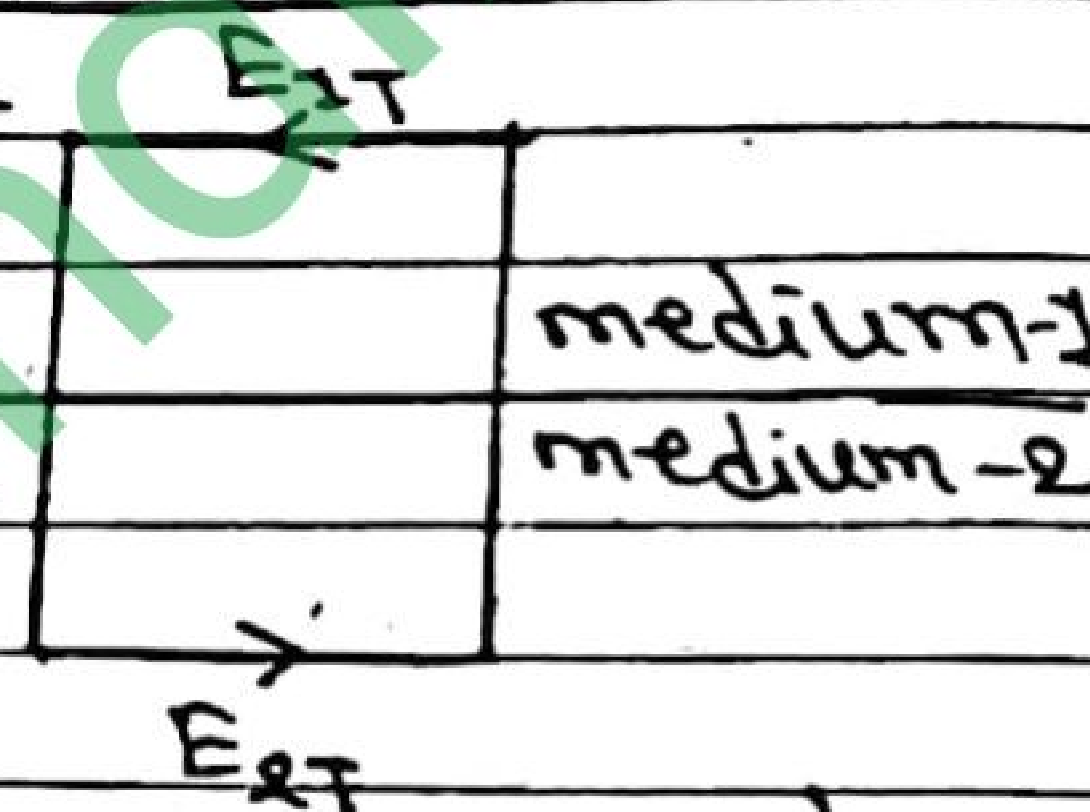
or, $\int D_{1n} ds = \int D_{2n} ds$

$\therefore D_{1n} = D_{2n}$

It shows that the normal component of electric displacement vector is continuous at the boundary.

Boundary condition for $\vec{E}$

Consider a loop elementary region connecting medium-1 and medium-2 as shown in figure.



Since the electric field is conservative in nature, we can write

$\oint \vec{E} dl$

or, $\int E_{1T} dl = \int E_{2T} dl = 0$

or, $\int E_{1T} dl = \int E_{2T} dl$

$\therefore E_{1T} = E_{2T}$

It shows that the tangential component of electric field vector is continuous at the boundary.