* Fluid Mechanics

Streamline flow (ordered flow, steady state flow, laminar flow)

If the velocity of fluid particles at every point is same in magnitude as well as in direction, the flow is called streamline flow. The velocity of streamline flow does not depend on time.

Turbulent flow

If the velocity of fluid particles is different in magnitude and direction is called turbulent flow. It is random flow.

Pressure Energy

The workdone against fluid pressure is stored in terms of energy called pressure energy.

If p be the pressure of fluid, 'a' be the cross-sectional area of tube in which fluid is flowing and 'x' is the displacement produced then pressure energy is given as,

$= Pax$

Pressure energy per unit mass

$$= \frac{pax}{\rho ax} = \frac{p}{\rho}$$

Where ρ be the density of fluid.

Kinetic energy

It is the energy possessed by fluid due to its motion. If m be the mass and v be the velocity of the fluid. Then the K.E. is

$$K.E. = \frac{1}{2}mv^2$$

and K.E per unit mass $= \frac{1}{2}v^2$

Potential energy

It is the energy possessed by fluid due its position relative to earth surface. If m be the mass of fluid, h be the ~~the~~ height of the fluid, then potential energy (P.E.) = mgh

Potential energy per unit mass

$= gh$

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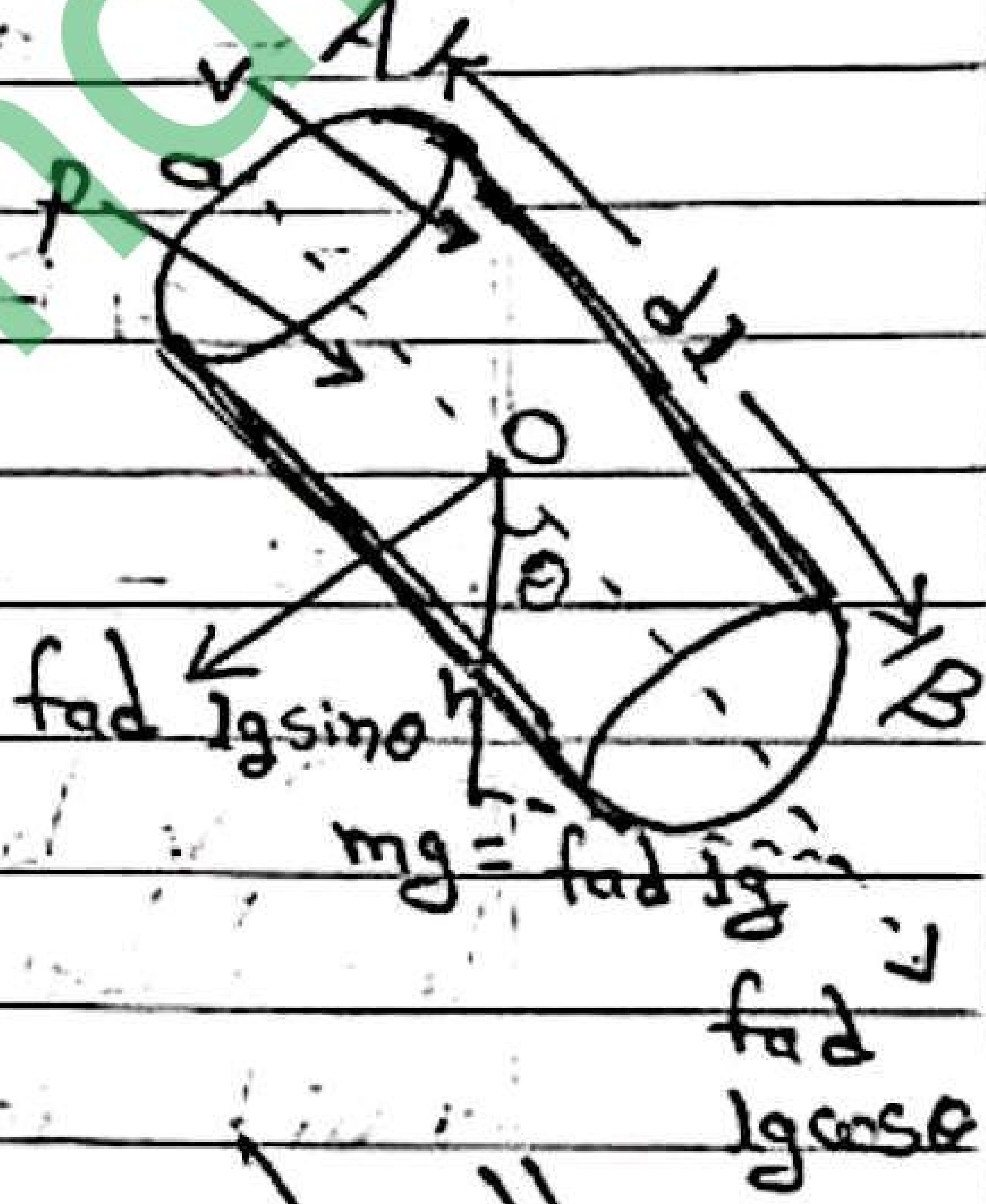
#. Bernoulli principles

The total energy per unit mass of the incompressible, non-viscous fluid in steady state flow remains constant throughout the flow.

i.e. $\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}$

Consider the section AB of tube of length dl and cross-sectional area 'a', density of the fluid inside the tube 'ρ'. So, mass of fluid inside the section AB = ρadl



The weight of fluid inside the section AB acts in downward direction from 'O'. θ be the angle between axis of the tube and the line along which the weight of the mass ρadl acts.
mg cosθ acts long the axis of the tube in downward direction.
If p be the pressure at end A in

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forward direction and $\frac{\partial P}{\partial l}$ be the change of pressure w.r.t. 'l' then the pressure at end B is $p + \frac{\partial P}{\partial l} dl$ in backward direction.

Now the net force in downward direction along the axis of the tube is

$$F = Pa - \left(p + \frac{\partial p}{\partial l} dl\right)a + \rho a dl g \cos\theta$$

$$\text{or, } F = -\frac{\partial p}{\partial l} dl\, a + \rho a dl g \cos\theta \longrightarrow (1)$$

If v be the velocity of the fluid inside the tube then the force acting in the fluid is

$$F = m\frac{dv}{dt}$$

Taking v as the explicit function of length and time.

i.e. $v = v(l, t)$

$$\text{or, } dv = \frac{\partial v}{\partial l} dl + \frac{\partial v}{\partial t} dt$$

or, $\dfrac{dv}{dt} = \dfrac{\partial v}{\partial l}\dfrac{dl}{dt} + \dfrac{\partial v}{\partial t}$

or, $\dfrac{dv}{dt} = \dfrac{v\partial v}{\partial l} + \dfrac{\partial v}{\partial t}$

$\therefore F = \rho a dl \left(\dfrac{v\partial v}{\partial l} + \dfrac{\partial v}{\partial t}\right) \longrightarrow (2)$

Equating (1) and (2), we get

$$\rho a dl \left(\dfrac{v\partial v}{\partial l} + \dfrac{\partial v}{\partial t}\right) = -\dfrac{\partial p}{\partial l} dl a + \rho a dl g \cos\theta \longrightarrow (3)$$

where $\cos\theta = -\dfrac{\partial h}{\partial l}$ and h is the height of point 'O' from any reference plane and $\cos\theta$ is taken -ve for downward flow, then eq^{n} (3) becomes,

$$\rho a dl \left(\dfrac{v\partial v}{\partial l} + \dfrac{\partial v}{\partial t}\right) = -\dfrac{\partial p}{\partial l} a - \rho a dl g \dfrac{\partial h}{\partial l}$$

or, $\left(\dfrac{v\partial v}{\partial l} + \dfrac{\partial v}{\partial t}\right)\rho = -\dfrac{\partial p}{\partial l} - \rho g \dfrac{\partial h}{\partial l} \longrightarrow (4)$

Eq^{n} (4) is called Euller's eq^{n} and is valid for both steady state as well as turbulent flow.

For steady state flow $\frac{\partial v}{\partial t} = 0$ and all partial derivatives reduces to total derivative.

$$\rho \frac{v\,dv}{dl} = -\frac{dP}{dl} - \rho g \frac{dh}{dl}$$

or, $\rho v\,dv + \frac{dP}{\rho} + \rho g\,dh = 0$

or, $v\,dv + \frac{dP}{\rho} + g\,dh = 0$

Integrating over streamline,

$$\int v\,dv + \frac{1}{\rho}\int dP + g\int dh = 0$$

$$\frac{v^2}{2} + \frac{P}{\rho} + gh = \text{cosnstant} \longrightarrow (5)$$

which is Bernoulli's theorem.

Multiplying eq^n (5) by ρ.

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant} \longrightarrow (6)$$

Each term of eq^n (6) has dimension of pressure. $(P + \rho gh)$ is static pressure & $\frac{1}{2}\rho v^2$ is dynamic pressure.

i.e. static pressure + dynamic pressure = constant → (7)

Which is other form of Bernoulli's theorem.
Dividing eqn (5) by 'g', we get

$$\frac{1}{2}\frac{v^2}{g} + \frac{P}{\rho g} + h = \text{constant} \longrightarrow (8)$$

Each term of eqn (8) has dimension of length (height) or head. First term is velocity head, 2nd term is pressure head and h is gravitational head.

∴ Pressure head + velocity head + gravitational head = constant ⟶ (9)

Which is next form of Bernoulli's principles.

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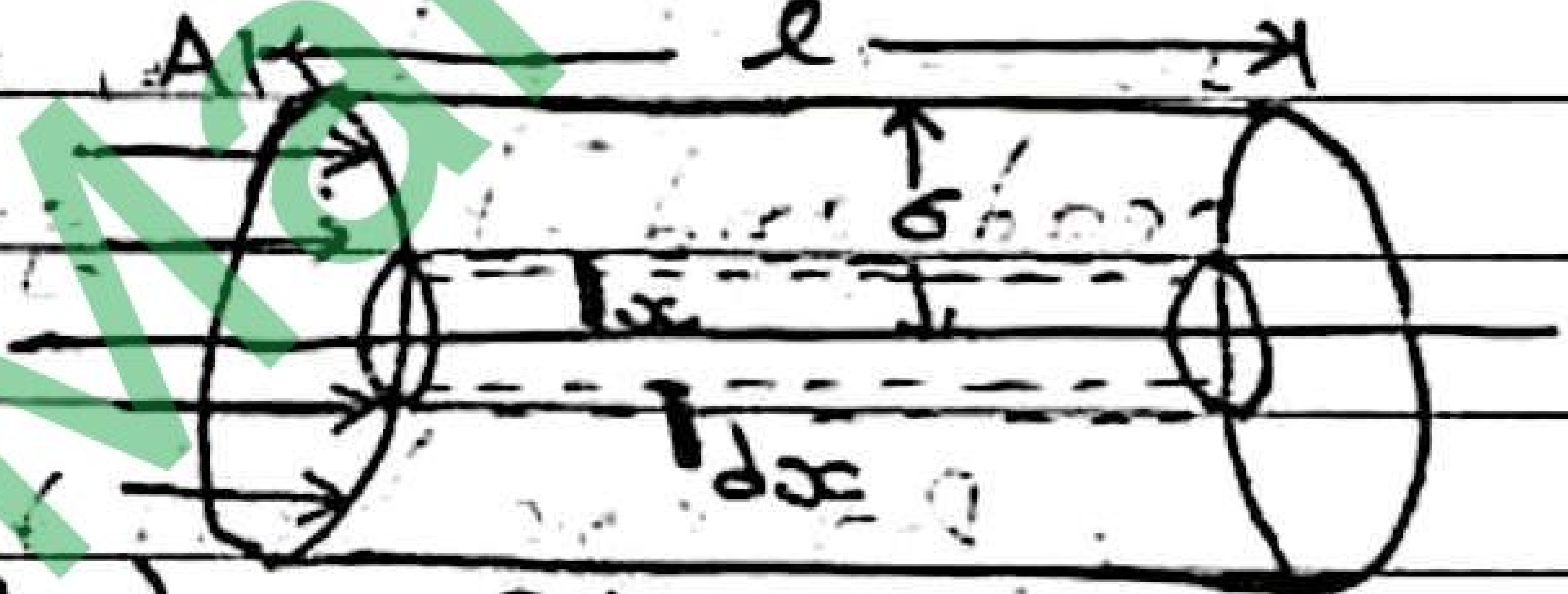
Poiseulle's Principles

Poiseulle's found an expression for the rate of fluid (liquid) flow through capillary tube considering following assumptions.

1. The flow of the fluid through tube is streamline.

2. The pressure at every point in each cross-sectional area of tube is same.
3. The velocity of the fluid is maximum at the axis, decreases gradually towards the walls of the tube and zero at the surface of the tube.
4. This formula is valid for very narrow tube, i.e. capillary tube.

Consider the section AB of the capillary tube whose



length 'l' and fluid is following from left to right. 'r' be the radius of the tube. The velocity of fluid is maximum at the axis and decrease towards the surface, and the tube is cylindrical. The flow of the fluid inside the tube is also cylindrical form. Out of these cylinder radius 'x' and thickness 'dx'. The backward dragging force i.e. viscous force acting on the surface area of the cylinder is given as:

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$$F = -\eta A \frac{dv}{dx}$$

Where η be the coefficient of viscosity of the fluid.

$A = 2\pi x l$

$$\therefore F = -2\pi\eta x l \frac{dv}{dx} \longrightarrow (1)$$

If p be the pressure difference between its two ends. Then the force acting in forward direction in cross-sectional area of cylinder is

$$F = P \times \pi x^2 \longrightarrow (2)$$

For steady state flow,

$$P \times \pi x^2 = -2\pi\eta x l \frac{dv}{dx}$$

Where -ve sign means two force are opposite in nature.

$$\therefore dv = \frac{-Px\,dx}{2\eta l}$$

By Integrating

$$\int dv = \int \frac{-Px\,dx}{2\eta l}$$

$$v = \frac{-P}{2\eta l}\left[\frac{x^2}{2}\right] + c$$

where v = velocity and c = Integration constant and evaluated using poiseulle's assumptions.

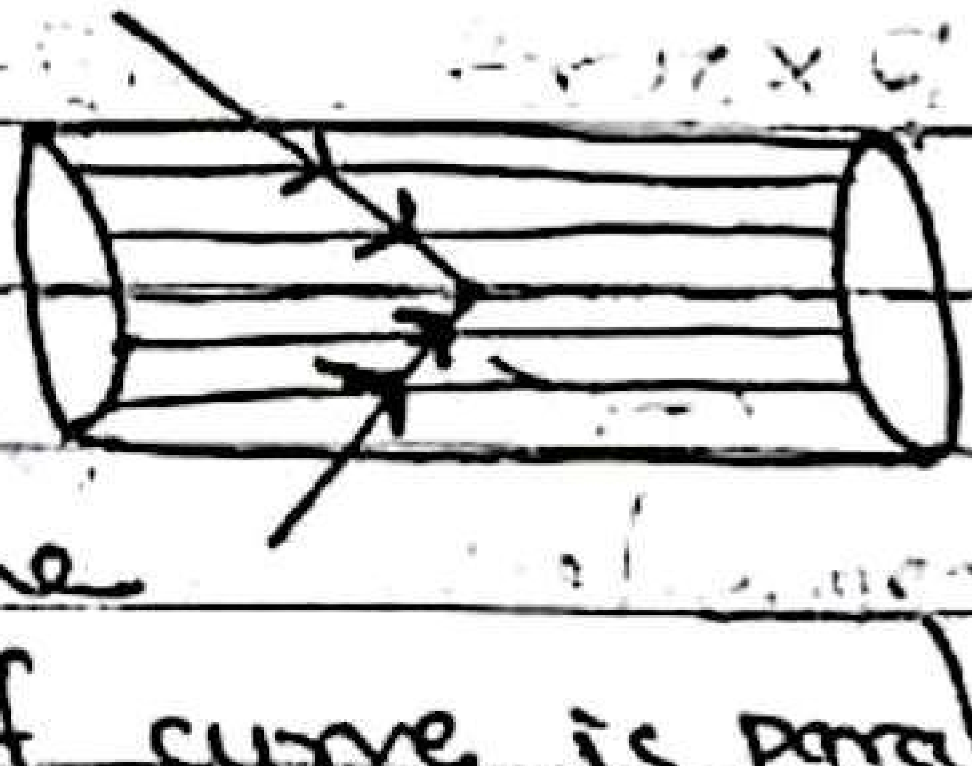
i.e. At $x = r$, $v = 0$

$$\therefore\ 0 = \frac{-Pr^2}{4\eta l} + c$$

$$c = \frac{Pr^2}{4\eta l}$$

$$\therefore\ v = \frac{P}{4\eta l}(r^2 - x^2) \longrightarrow (3)$$

Which gives the velocity of fluid at any distance x from centre of the



tube. The velocity of curve is parabolic in nature. Now the volume of fluid flowing per second through cross-sectional area of the cylinder of radius 'x' and thickness 'dx' is

$$dq = 2\pi x\,dx\,v$$

or, $$dQ = 2\pi x\,dx \times \frac{P}{4\eta l}(r^2 - x^2)$$

or, $$dq = \frac{2\pi P}{4\eta l}(r^2 - x^2)\,x\,dx$$

The flow of fluid through the capillary tube is given by

$$Q = \int dq = \frac{2\pi P}{4\eta l}\int_0^r (r^2 - x^2)\,x\,dx$$

or, $$Q = \frac{2\pi P}{4\eta l}\left(\frac{r^4}{2} - \frac{r^4}{4}\right) = \frac{2\pi P}{4\eta l} \times \frac{r^4}{4}$$

$$\therefore Q = \frac{\pi P r^4}{8\eta l}$$

This gives the volume of fluid flowing per second through tube.

Limitations

1. This formula holds only for streamline flow of the liquid.
2. It is not true of the layer's of the liquid in contact of the walls of the tube are not at rest and thus have a relative slipping.

3. Poiseuille's formula has been derived on the basis that the pressure difference across the tube is used to overcome the viscous forces and no part of it is spent in giving K.E. to the liquid. Thus this formula is true for liquid velocities involving negligible amount of K.E.

Reynolds's number:

Reynolds's studies the motion of fluid in which both inertial force and viscous force are operative and found that the ratio of inertial force to the viscous force is constant, called Reynolds's number. we have the expression for critical velocity V_c is given by

$$V_c = \frac{K\eta}{\rho r} \longrightarrow (1)$$

where, η = coeff. of viscosity of fluid, ρ = density of the fluid, r = radius of the tube.

Below critical velocity, the motion is streamline and beyond the critical

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velocity motion is turbulent.
Now from eqn. (1)

$$K = \frac{\rho r v_c}{\eta} = \frac{\rho v_c^2}{\eta \frac{v_c}{r}}$$

$$\text{or, } K = \frac{m}{A \times d} \times \frac{\left(\frac{d}{t}\right)^2}{\eta \frac{v_c}{r}}$$

$$\text{or, } K = \frac{\frac{m}{d} \times \left(\frac{d}{t}\right)^2}{\eta A \frac{v_c}{r}}$$

$$\text{or, } K = \frac{\text{mass} \times \text{accleration}}{\text{Viscous force}}$$

$$\therefore K = \frac{\text{Inertial force}}{\text{Viscous force}}$$

which is called Reynold's number

If $K < 2000$, the flow is streamline.
If $K > 3000$, the flow is turbulent.

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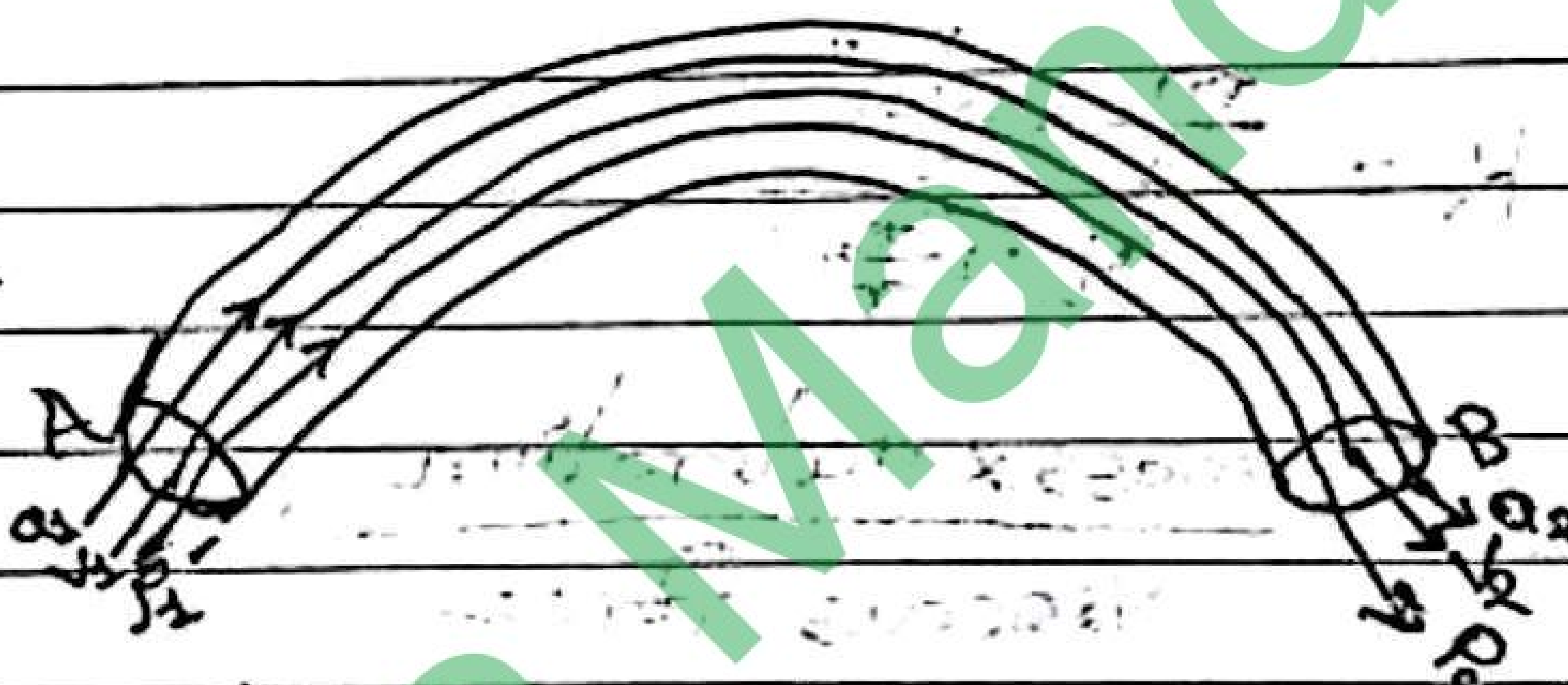
Equation of continuity in fluid dynamics

The volume rate of flow of incompressible fluid remains constant through any section of the tube through its motion.



Consider a section AB of tube in which fluid is flowing from end A to end B. a_1 and a_2 be the cross-sectional area of end A and B resp. v_1 and v_2 be the velocities of fluid entering and outgoing from the section AB. ρ_1 be the density of fluid entering from A and ρ_2 be the density of fluid outgoing from B.

Now, using conservation of mass, we have
Mass of the fluid entering from end

A per sec = mass of fluid out going from end B per sec

i.e. $\rho_1 a_1 v_1 = \rho_2 a_2 v_2$

For incompressible fluid $\rho_1 = \rho_2 = \rho$

$\therefore a_1 v_1 = a_2 v_2$

$\Rightarrow av = \text{constant}$

$\therefore v \propto \frac{1}{a}$

Which is called eq^n of continuity.

Capillaries in series and Parallel

> In series

Let n be the no. of capillary tubes of length $l_1, l_2 --- l_n$ and radii $r_1, r_2 --- r_n$ be connected in series. Let a liquid of coefficient of viscosity η flow through them in streamline flow. Q be the rate of liquid flow through each tube and the pressure difference across this tube be $P_1, P_2 --- P_n$ and across the combination as we whole, P so that

$P = P_1 + P_2 + --- + P_n$

If $R_1, R_2 --- R_n$ be the effective

Viscous resistance for the n tubes and R for the combination, we have

$$R_1 = \frac{P_1}{Q}$$

$$R_2 = \frac{P_2}{Q}$$

$$R_n = \frac{P_n}{Q}$$

$$\therefore R_1 + R_2 + \dots + R_n = \frac{1}{Q}(P_1 + P_2 + \dots + P_n)$$

$$= \frac{P}{Q} \longrightarrow (1)$$

But, $R = \frac{P}{Q} \longrightarrow (2)$

From eqn (1) and (2), we get

$R = R_1 + R_2 + \dots + R_n$

Now, $Q = \frac{\pi P r^4}{8 L \eta}$

$$\therefore R = \frac{8\eta}{\pi}\left[\frac{L_1}{r_1^4} + \frac{L_2}{r_2^4} + \dots + \frac{L_n}{r_n^4}\right]$$

→ In parallel

Let the number of capillary tube 'n' of length $l_1, l_2 --- l_n$ and radii $r_1, r_2 --- r_n$ lying in the same horizontal plane and parallel with each other. η be the coefficient of viscosity of liquid. The pressure difference across each capillary will be same, say P. Let the rate of liquid flow through the capillary tubes be $Q_1, Q_2 --- Q_n$ resp. Then, the rate of flow of liquid through combination,

$$Q = Q_1 + Q_2 + --- + Q_n$$

$$= \left[\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2} + --- + \frac{\pi r_n^4}{8\eta l_n}\right.$$

Let $R_1, R_2 --- R_n$ be the effective viscous resistance for the tubes and R be the total effective resistance of the combination, Then

$$\frac{P}{R} = \frac{P}{R_1} + \frac{P}{R_2} + --- + \frac{P}{R_n}$$

Since, $Q_1 = \frac{P}{R_1}$ --- $Q_n = \frac{P}{R_n}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + --- + \frac{1}{R_n}$$

Thus, the reciprocal of total effective viscous resistance of liquid for the combination is equal to the sum of effective viscous resistance for the number of capillary tube.