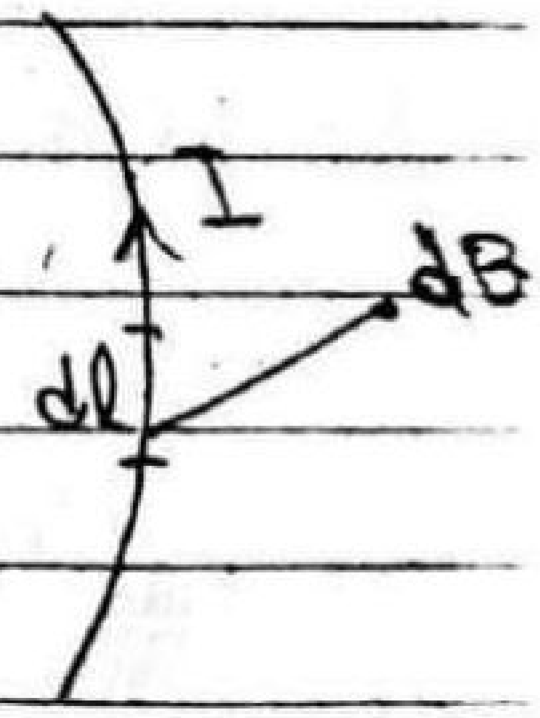
chapter 20

Magnetic field on moving Charge

Biot-Savart law

Biot and savart law is a mathematical expression giving the value of small magnetic field produced by the flow of current in the length element of conductor. If I be the current flows, length of element 'dl', the small magnetic field produced will be

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$



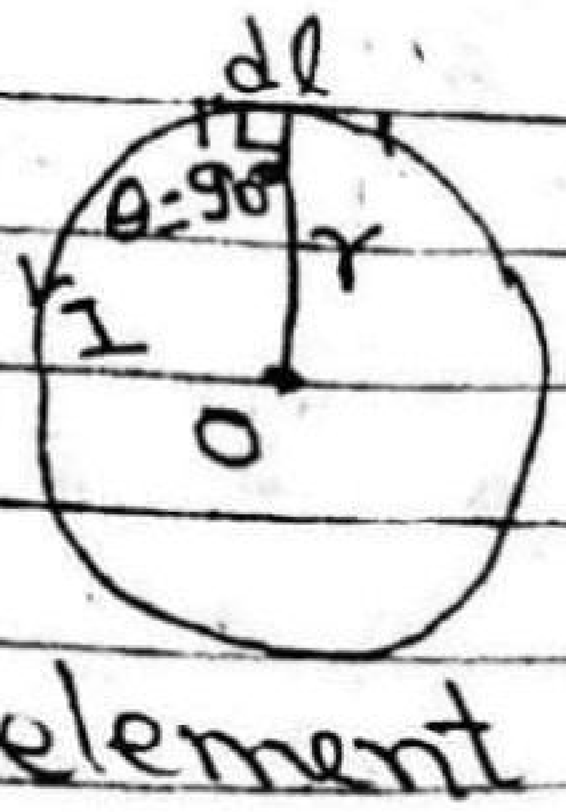
In vector form

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{(d\vec{l} \times \vec{r})}{r^3}$$

→ Applications of Biot-savart law

1. Magnetic field at the centre of circular coil

Let us consider a circular coil of radius r and carrying current I. Take a small element of length



'dl' on the coil as shown in figure. Then, According to Biot-savart law the small magnetic field due to current in 'dl' at the centre will be

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

When $\theta = 90^\circ$ then

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \longrightarrow (1)$$

The total magnetic field is obtain by integrating the above eq^n (1) within limit 0 to $2\pi r$

$$\text{So, } B = \int_0^{2\pi r} \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_0^{2\pi r} dl$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} \left[2\pi r \right]$$

$$\boxed{\therefore B = \frac{\mu_0}{2} \frac{I}{r}}$$

If the coil has N term of wire the total magnetic field produced will be

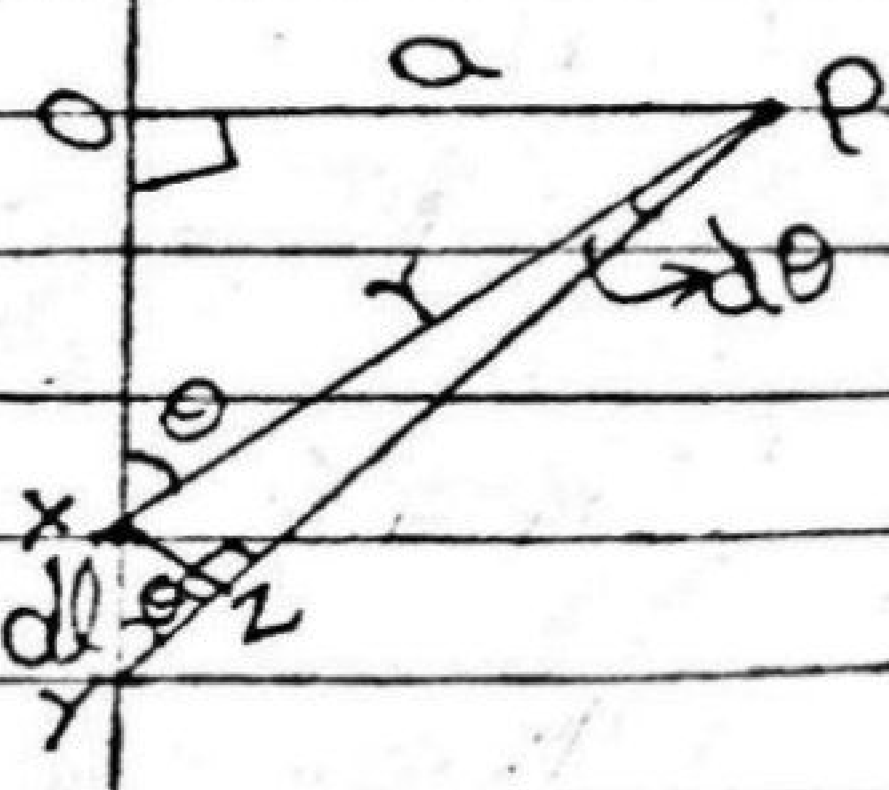
$$B_N = N \frac{\mu_0 I}{2r}$$

Direction of magnetic field produced

According to right hand grip rule the direction of magnetic field produced is along the axis of coil. In the case as shown in figure the direction of magnetic field is perpendicularly outward.

Q/2. **Magnetic field produced by a long straight wire**

Let us consider a long straight wire carrying current I. Take a point P at distance 'a' from a wire. In order to use Biot-Savart law take a small length element 'dl' as shown in figure.



Here

$\angle OXP \approx \angle OYP = \theta$ and

$\angle XPY = d\theta$

draw $XZ \perp YP$

According to Biot-Savart law the small magnetic field produced at p due to current in 'dl' is given by

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2} \longrightarrow (1)$$

From rt. angle ΔOXP, then

$$\sin\theta = a/r$$

$$\therefore r = a/\sin\theta \longrightarrow (2)$$

Also, From rt. angle ΔXYZ

$$\sin\theta = \frac{XZ}{dl}$$

$$\therefore dl = \frac{XZ}{\sin\theta} \longrightarrow (3)$$

Now from rt. angle ΔXZP, then

$$\sin d\theta = \frac{XZ}{r}$$

When $\sin d\theta \approx d\theta$,

So $d\theta = \frac{XZ}{r}$

$\therefore XZ = r d\theta \longrightarrow (4)$

From eqn. (3) and (4), we have

$$dl = \frac{r d\theta}{\sin\theta}$$

Now, eqn. (1) becomes

$$dB = \frac{\mu_0}{4\pi} I \frac{\frac{r d\theta}{\sin\theta} \sin\theta}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{d\theta}{r}$$

$$= \frac{\mu_0 I}{4\pi} \frac{d\theta}{(a/\sin\theta)}$$

$$= \frac{\mu_0 I}{4\pi a} \sin\theta \, d\theta$$

If θ_1 and θ_2 are the angles made by 'r' with ends of wire then total magnetic field will be.

$$B = \int_{\theta_1}^{\theta_2} \frac{\mu_0 I \sin\theta \, d\theta}{4\pi a}$$

$$= \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= \frac{\mu_0 I}{4\pi a}\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\therefore \boxed{B = \frac{\mu_0 I}{4\pi a}\left(\cos\theta_1 - \cos\theta_2\right)} \longrightarrow (5)$$

This eqn (5) gives the magnetic field produced by a straight wire of finite length.

For the infinitely long wire
$\theta_1 = 0$, $\theta_2 = \pi$ then

$$B = \frac{\mu_0 I}{4\pi a}\left(\cos 0 - \cos\pi\right)$$

$$= \frac{\mu_0 I}{4\pi a}\left(1 - (-1)\right)$$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi a}} \longrightarrow (6)$$

This is eqn (6) gives the magnetic field produced by the infinitely long wire.

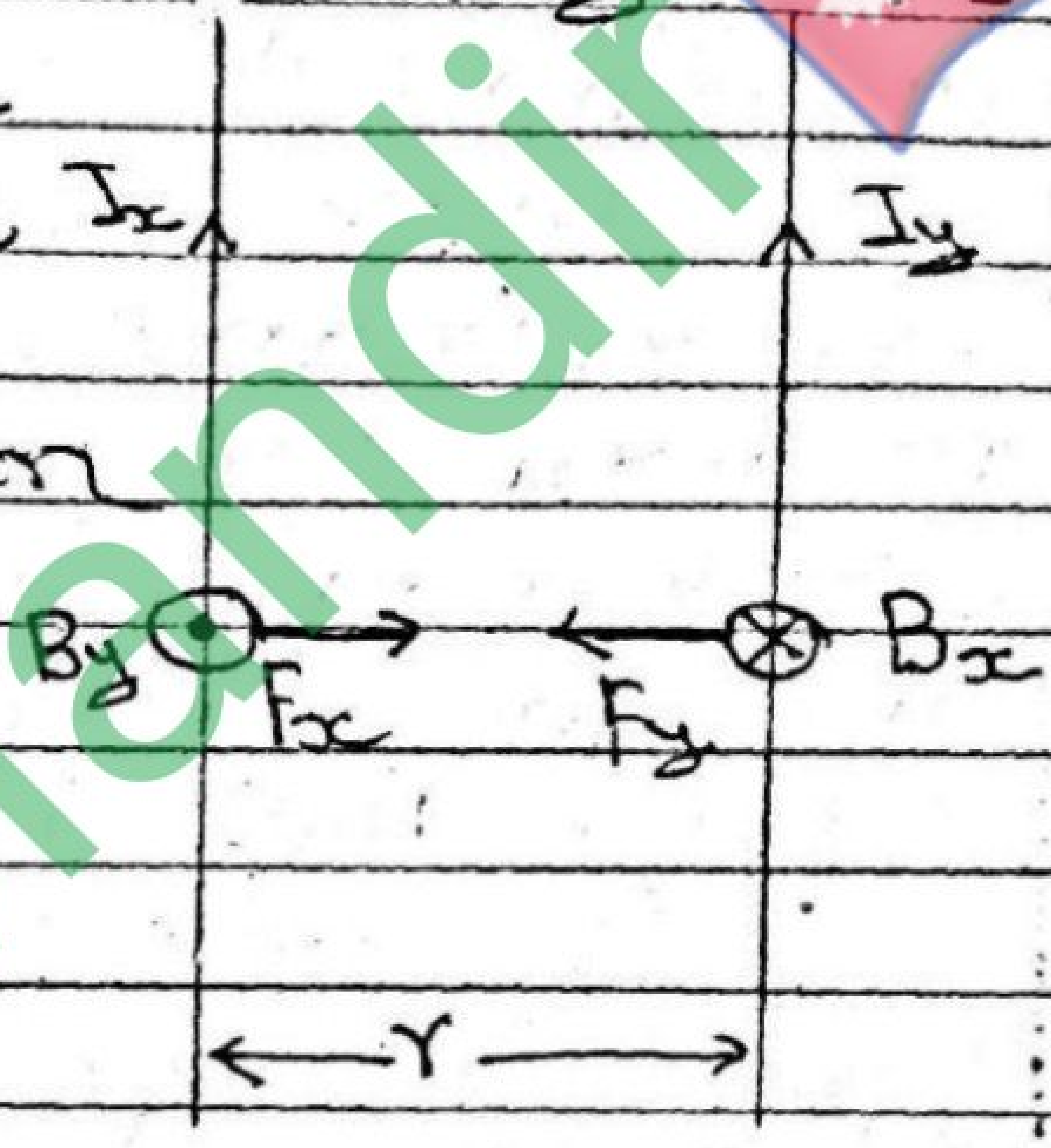
<u>Direction of magnetic field</u>

According to right hand thumb rule the magnetic field produced has direction prependicular to the plane of paper.

In this case as shown in figure, it is acting perpendicularly inward.

Force between two parallel wires carrying current.

Consider two wire x and y are carrying current I_x and I_y in the same direction. Let r be the distance between two parallel wires as shown in figure.



Here, Magnetic field produced by wire 'x' at distance r from it is given by

$$B_x = \frac{\mu_0 I_x}{2\pi r}$$

The wire y in this magnetic field so it experiences a force is

$$F_y = B_x I_y L_y \sin 90$$

$$\text{or, } F_y = B_x I_y L_y = \frac{\mu_0 I_x I_y L_y}{2\pi r}$$

So, force per unit length becomes

$$f_y = \frac{F_y}{L_y}$$

or $f_y = \frac{\mu_0 I_x I_y}{2\pi r}$

Similarly, the wire x in this magnetic field so it experiences force per unit length is $f_x = \frac{\mu_0 I_x I_y}{2\pi r}$

According to fleming's left hand rule, these forces are acting towards the respective wire. so the wires are attracted towards each other.

In general, the force per unit length on both wires will be

$$f = \frac{\mu_0 I_x I_y}{2\pi r}$$

Q-> Find current carrying by two parallel wires which produced a force betn. them of 2×10^{-7} N/m when placed 1m apart.

=> Solution,

For per unit length (f) = 2×10^{-7} N/m

distance (r) = 1m

current ($I_1 = I_2$) = ?

we have,

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$\odot \rightarrow$ current direction (outward)
$\otimes \rightarrow$ current direction (Inward)

or, $2\times10^{-7} = \dfrac{4\pi\times10^{-7}\, I_1\times I_1}{2\pi\times1}$

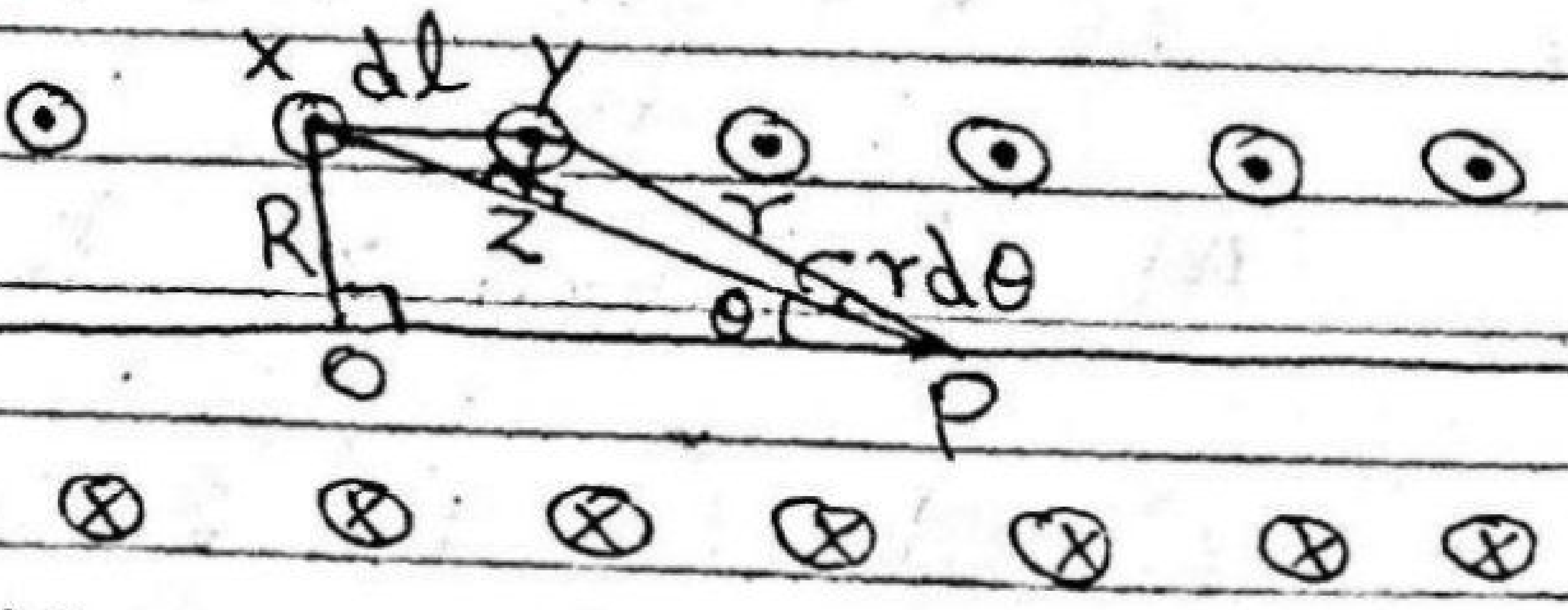
or, $I_1^2 = 1A$

$\therefore I_1 = I_2 = 1A.$

* <u>Note</u>: 1A is the equal current flowing in two parallel wires separated by 1m distance if the force per unit length on them is equal to 2×10^{-7} N/m.

(I) 3. <u>Magnetic field on the axis of current carrying solenoid</u>

Consider a solenoid having no. of turns per unit length 'n' and carrying current 'I'. Take



a small elementary of solenoid with length dl and no. of turns 'N'. The small magnetic field produced by this small portion at the point 'p' is given by

$$dB = \frac{\mu_0 N I R^2}{2R^3}$$

or, $dB = \frac{\mu_0 n dl I R^2}{2r^3} \longrightarrow (1)$

where $n = N/dl$

Draw YZ perp. on XP. Since dl is very small we can write $YP \approx XP = r$.

From rt. ΔXOP

$\sin\theta = \frac{R}{r}$

$\therefore r = \frac{R}{\sin\theta} \longrightarrow (2)$

From rt. ΔXYZ

$\sin\theta = \frac{YZ}{dl}$

$dl = \frac{YZ}{\sin\theta} \longrightarrow (3)$

Also, from rt. ΔYZP then

$\sin d\theta = \frac{YZ}{r}$ or, $d\theta = \frac{YZ}{r}$ $(\sin\theta \approx d\theta)$

or, $YZ = r\,d\theta \longrightarrow (4)$

From eqn. (3) and (4)

$dl = \frac{r\,d\theta}{\sin\theta}$

Now, Eqn. (1) becomes,

$dB = \frac{\mu_0 n \frac{r\,d\theta}{\sin\theta} I R^2}{2r^3}$

$$= \frac{\mu_0 n I R^2 d\theta}{2 r^2 \sin\theta}$$

$$= \frac{\mu_0 n I R^2 d\theta}{2 \frac{R^2}{\sin^2\theta} \sin\theta}$$

$$= \frac{\mu_0 n I \sin\theta R^2 d\theta}{2R^2}$$

$$= \frac{\mu_0 n I \sin\theta d\theta}{2}$$

If θ_1 and θ_2 be the angles made by r with the ends of solenoid the total magnetic field produced will be

$$B = \int_{\theta_1}^{\theta_2} \frac{\mu_0 n I \sin\theta d\theta}{2}$$

$$= \frac{\mu_0 I n}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= \frac{\mu_0 I n}{2} \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\boxed{\therefore B = \frac{\mu_0 I n}{2} (\cos\theta_1 - \cos\theta_2)}$$

This equation gives the magnetic field ~~be~~ produced by a solenoid of finite length.
For infinitely long solenoid,
$\theta_1 = 0$ and $\theta_2 = \pi$

So $B = \frac{\mu_0 n I}{2}(\cos 0^\circ - \cos \pi)$

$= \frac{\mu_0 n I}{2}(1-(-1))$

$= \frac{\mu_0 n I}{2} \times 2$

$\therefore B = \mu_0 n I$

Direction of field
According to right hand grip rule the magnetic field produced by solenoid has direction along its axis.

Ampere circuital law
Ampere's circuital law is the alternative of Biot-Savart law. According to this law, the line integral of magnetic field along a closed path is equal to μ_0 times total enclosed by the path.

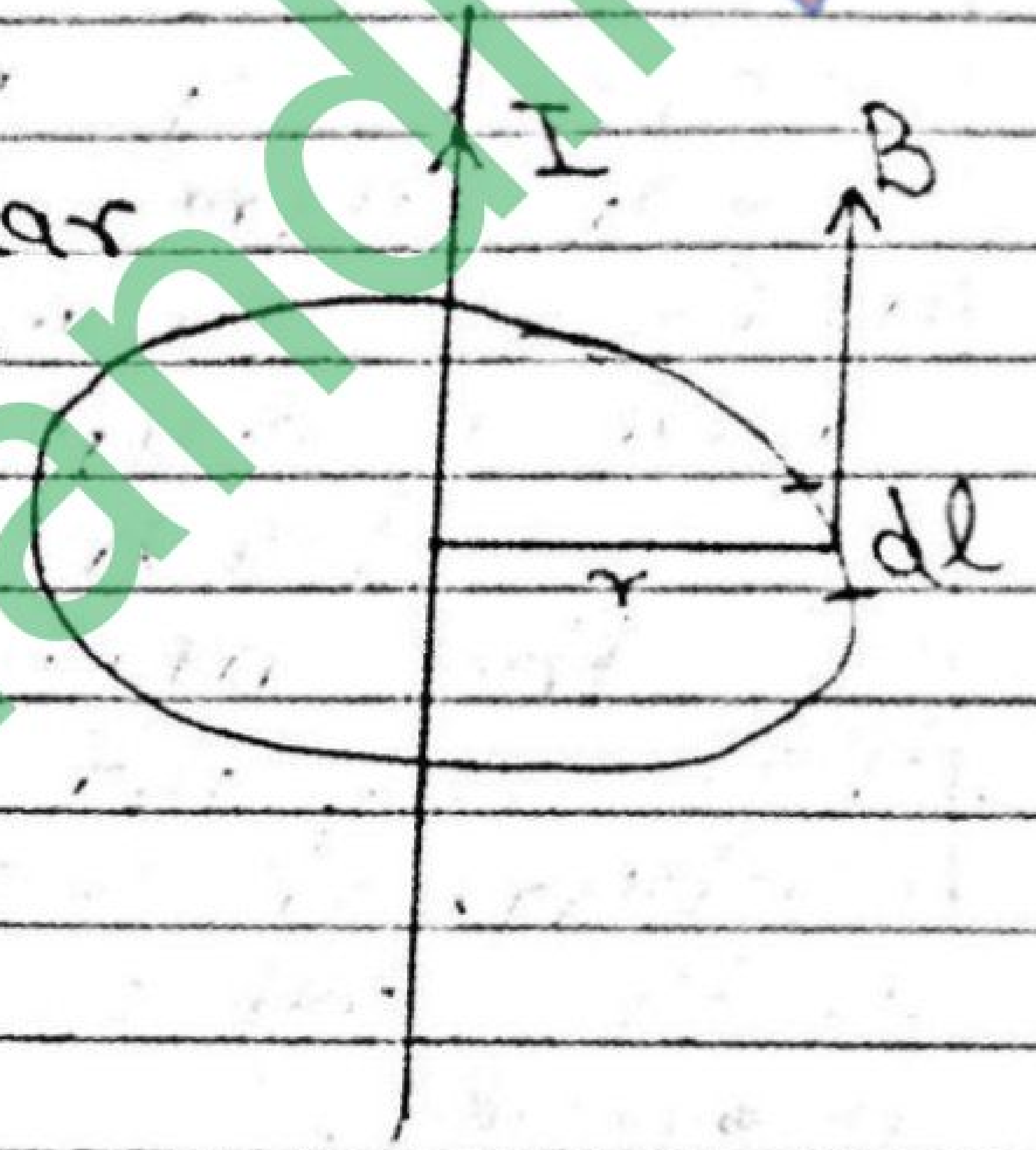
i.e. $\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I}$

Proof:

Suppose a long straight wire is carrying current 'I'. The magnetic field produced by this wire at distance r from it is given by,

$$B = \frac{\mu_0 I}{2\pi r}$$

Consider a circular closed path of radius 'r' with the wire along the axis as shown in figure.



Here,

$$\oint \vec{B} \cdot \vec{dl} = \oint B\,dl$$

$$= \oint \frac{\mu_0 I}{2\pi r}\,dl$$

$$= \frac{\mu_0 I}{2\pi r} \oint dl$$

$$= \frac{\mu_0 I}{2\pi r} \times 2\pi r$$

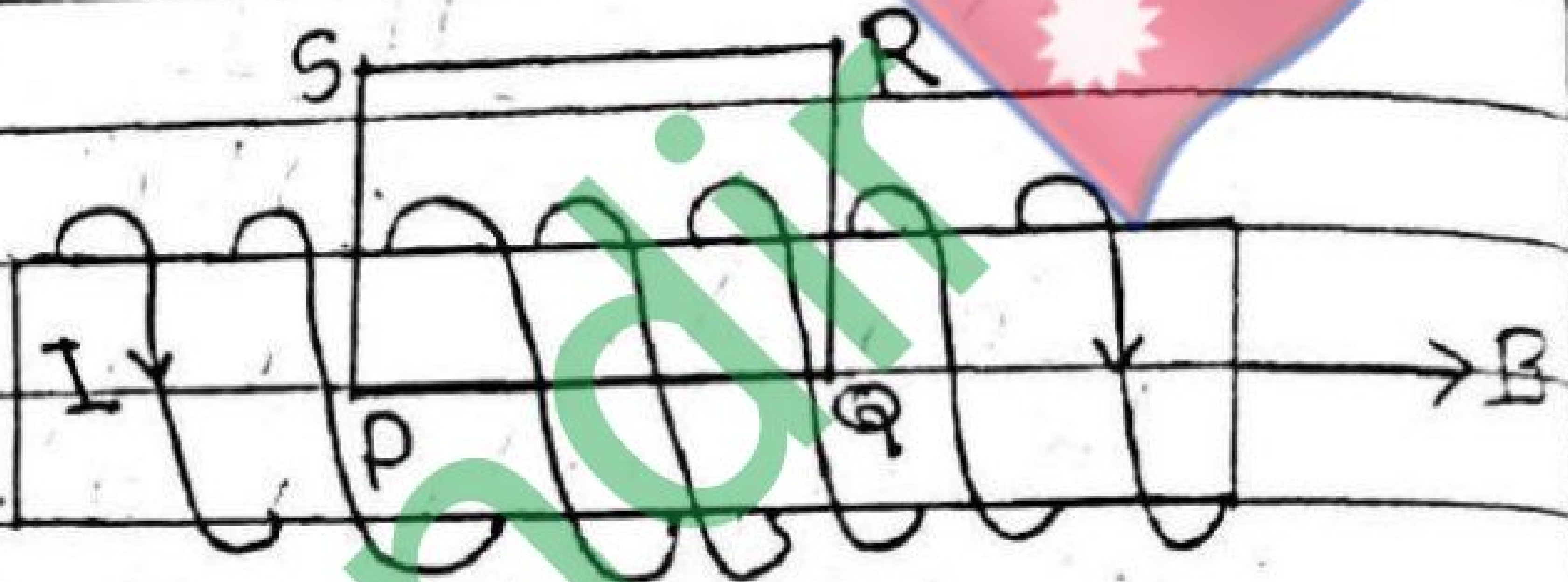
$$= \mu_0 I$$

$$\therefore \boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I} \longrightarrow (1)$$

This equation (1) gives the ampere's circuital law.

Applications of Ampere's circuital law

1. Magnetic field produced by current carrying solenoid



Consider a solenoid of 'n' no. of turns per unit length and carrying current I as shown in figure.

The magnetic field produced by this solenoid is strong and uniform within the coils and the field outside of the coil is weak and can be neglected.

Inorder to use Ampere's circuital law draw an imaginary rectangular path PQRS with PQ = l. If this imaginary path contains N turns of the solenoid then from Ampere's law,

$$\oint \vec{B}\,\vec{dl} = \mu_0 N I$$

or, $$\int_P^Q \vec{B}\,\vec{dl} + \int_Q^R \vec{B}\,\vec{dl} + \int_R^S \vec{B}\cdot\vec{dl} + \int_S^P \vec{B}\cdot\vec{dl} = \mu_0 N I$$

or, $\int_{P}^{Q} Bdl + 0 + 0 + 0 = \mu_0 NI$

or, $B\int_{P}^{Q} dl = \mu_0 NI$

or, $B.l = \mu_0 NI$ [$\because PQ = l$]

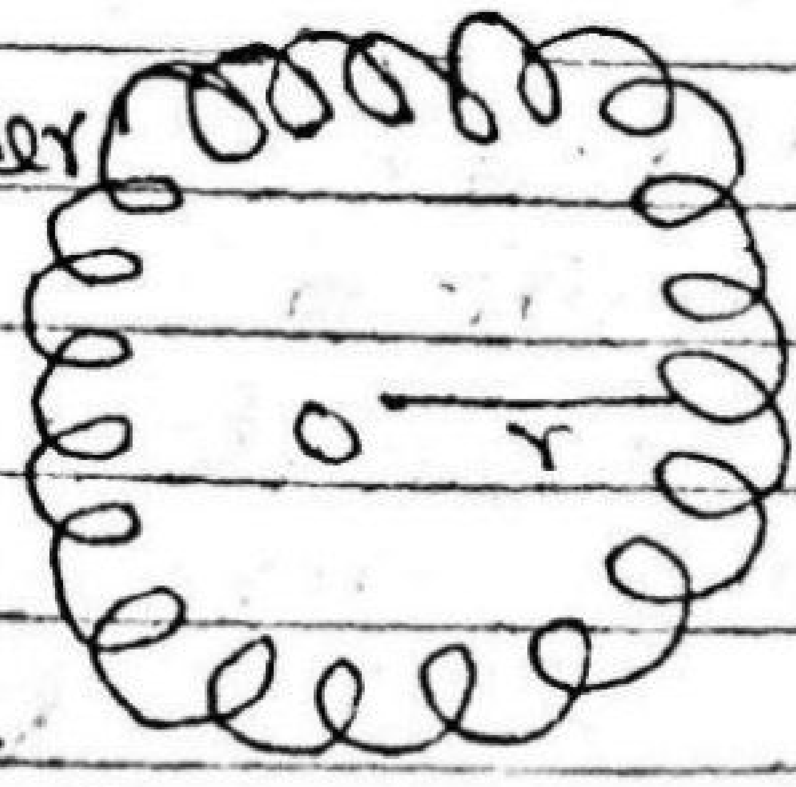
or, $B = \frac{\mu_0 NI}{l}$

$\therefore \boxed{B = \mu_0 n I}$ [$\because n = \frac{N}{l}$]

This is the required magnetic field produced by solenoid.

2. Magnetic field produced by current carrying toroid

Toroid is a circular ring of solenoid. consider a toroid having N no. of turns and carrying current to I as shown in figure.



In order to use Ampere's law, draw an imaginary circular closed path of radius r and passing through the axis of coil. From Ampere's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 N I$$

or, $\oint B\,dl = \mu_0 N I$

or, $B \oint dl = \mu_0 N I$

or, $B \cdot 2\pi r = \mu_0 N I$

or, $B = \dfrac{\mu_0 N I}{2\pi r}$

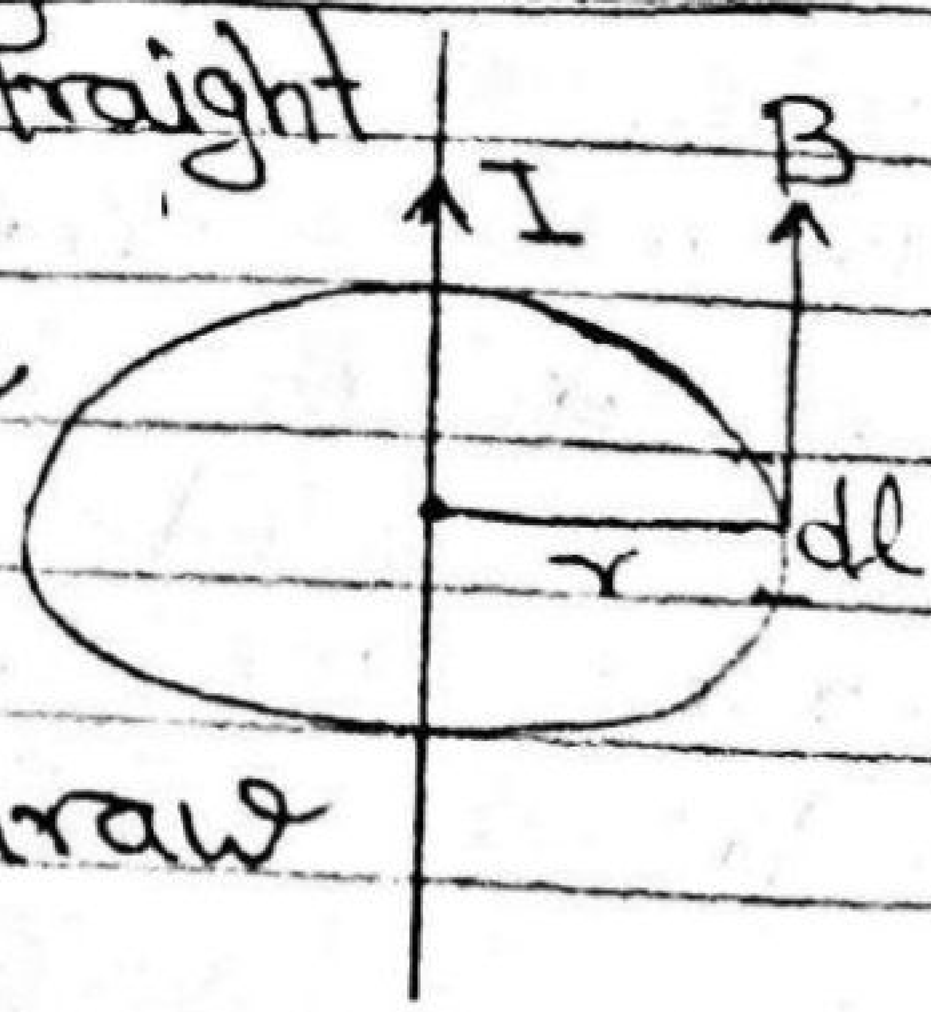
$\therefore B = \mu_0 n I$ where, $n = \dfrac{N}{2\pi r}$

This is the required magnetic field produced by current carrying toroid.

3. Magnetic field produced by current carrying long straight wire.

Consider a long straight wire carrying current I as shown in figure.

In order to find the magnetic field at distance r from it, draw



an imaginary circular closed path of radius r and containing the wire along the axis.
From Ampere's circuital law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\oint B\,dl = \mu_0 I$$

or, $B \cdot 2\pi r = \mu_0 I$

or, $\boxed{B = \frac{\mu_0 I}{2\pi r}}$

This is the required magnetic field produced by current carrying long straight wire.

Magnetic moment (μ) / Magnetic dipole moment

The magnetic moment of a current carrying loop is the product of current flowing in the loop and area of that loop. i.e.

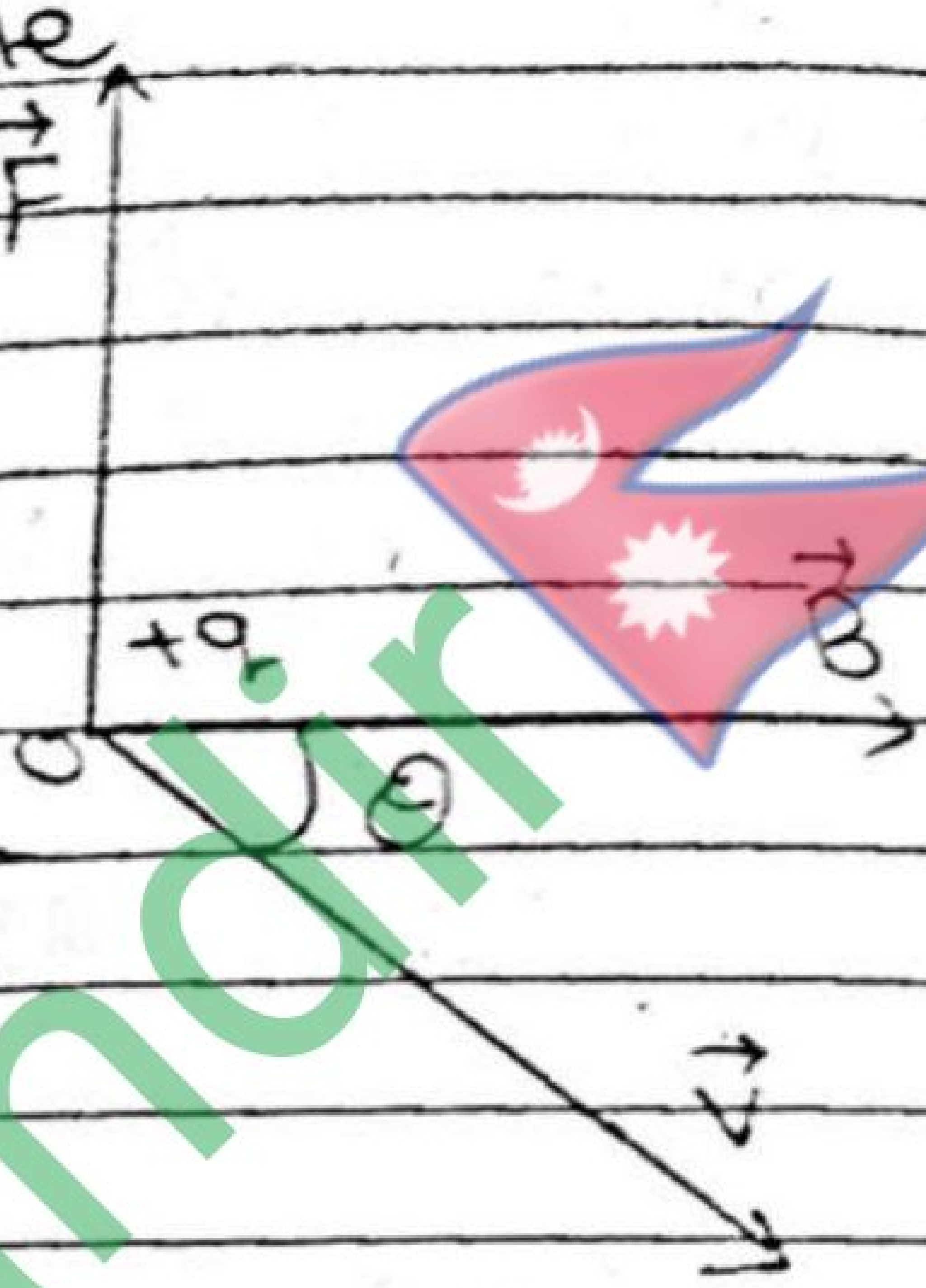
$\mu = IA$

The unit of magnetic moment is Am^2.

Lorentz Force

When a charge particle moves in a magnetic field it experiences a force. This force is called Lorentz force.

The magnitude of Lorentz force on a moving charge is given by



$$F = Bqv \sin\theta$$

Where,
B = magnetic field
q = amount of charge
v = speed with the moving charge

If a wire carrying current I is placed in the magnetic field the Lorentz force experienced by wire becomes,

$$F = BIl\sin\theta$$

The direction of Lorentz force is determined by the Fleming left hand rule.

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