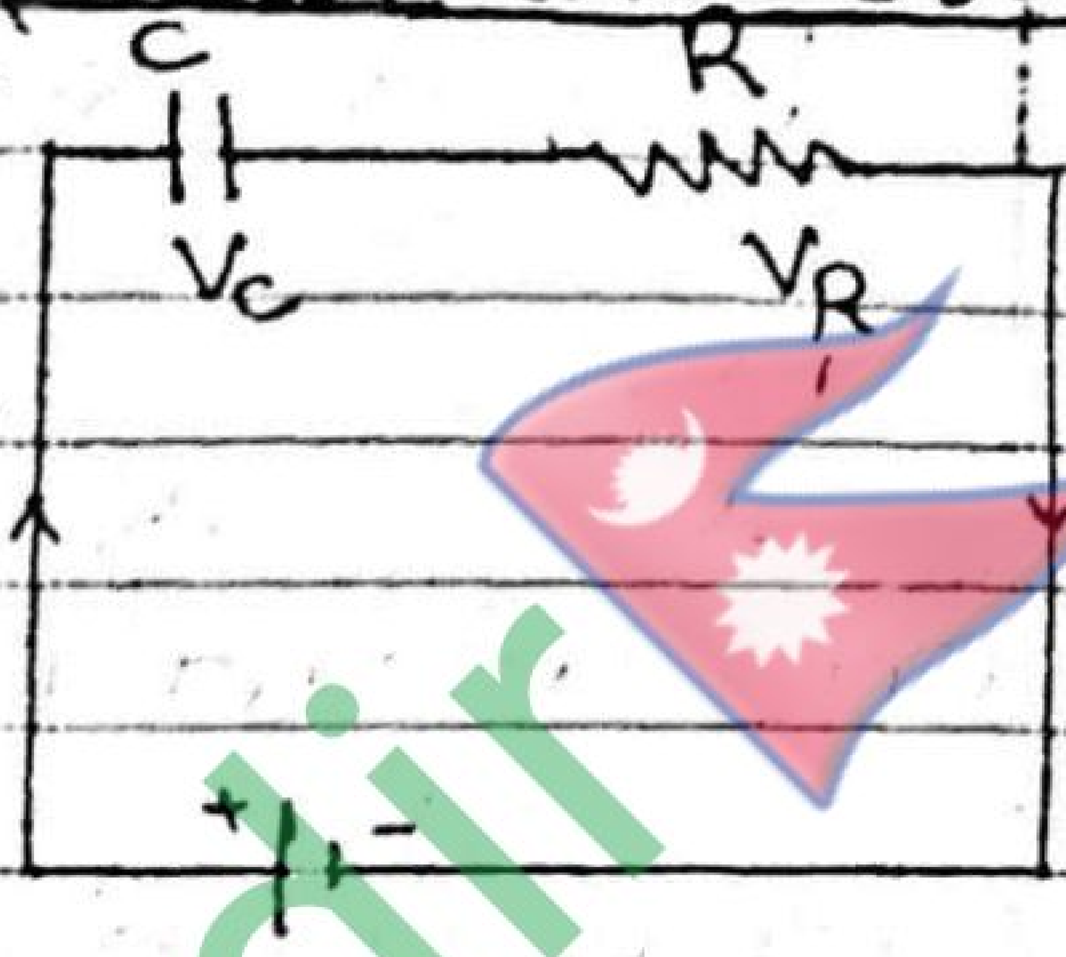
chapter 23

Varying Current

Charging of a capacitor (Resistor)

Suppose a capacitor of capacitance c is being charged by a battery of voltage V through a resistor of resistance R.



Here,

$V = V_c + V_R$

or, $V = \frac{q}{c} + IR$

or, $V = \frac{q}{c} + \frac{dq}{dt}R \quad [\because I = \frac{dq}{dt}]$

or, $V - \frac{q}{c} = \frac{dq}{dt}.R \longrightarrow (1)$

Let $V - \frac{q}{c} = x$, so,

$0 - \frac{1}{c}\frac{dq}{dt} = \frac{dx}{dt}$

or, $\frac{dq}{dt} = -\frac{dx}{dt}.c$

Now eq^n (1) becomes,

$V - \frac{q}{c} = -\frac{dx}{dt}c.R$

or, $\frac{dx}{dt} = -\frac{1}{CR}x$

or, $\frac{dx}{x} = -\frac{1}{CR}dt$

On integrating

$\ln x = -\frac{1}{CR}t + K$

or, $x = e^{(-\frac{1}{CR}t + K)}$

or, $x = e^{-\frac{1}{CR}t} \cdot e^{K}$

or, $x = Ae^{-\frac{t}{CR}}$ [where, $A = e^{K}$]

or, $V - \frac{q}{C} = Ae^{-\frac{t}{CR}}$ ⟶ (2) (using the value of x)

At $t = 0$, $q = 0$

$V - 0 = Ae^{0}$

or, $V = A$

Hence, eqn. (2) becomes,

$V - \frac{q}{C} = Ve^{-\frac{t}{CR}}$

or, $\frac{q}{C} = V - Ve^{-\frac{t}{CR}}$

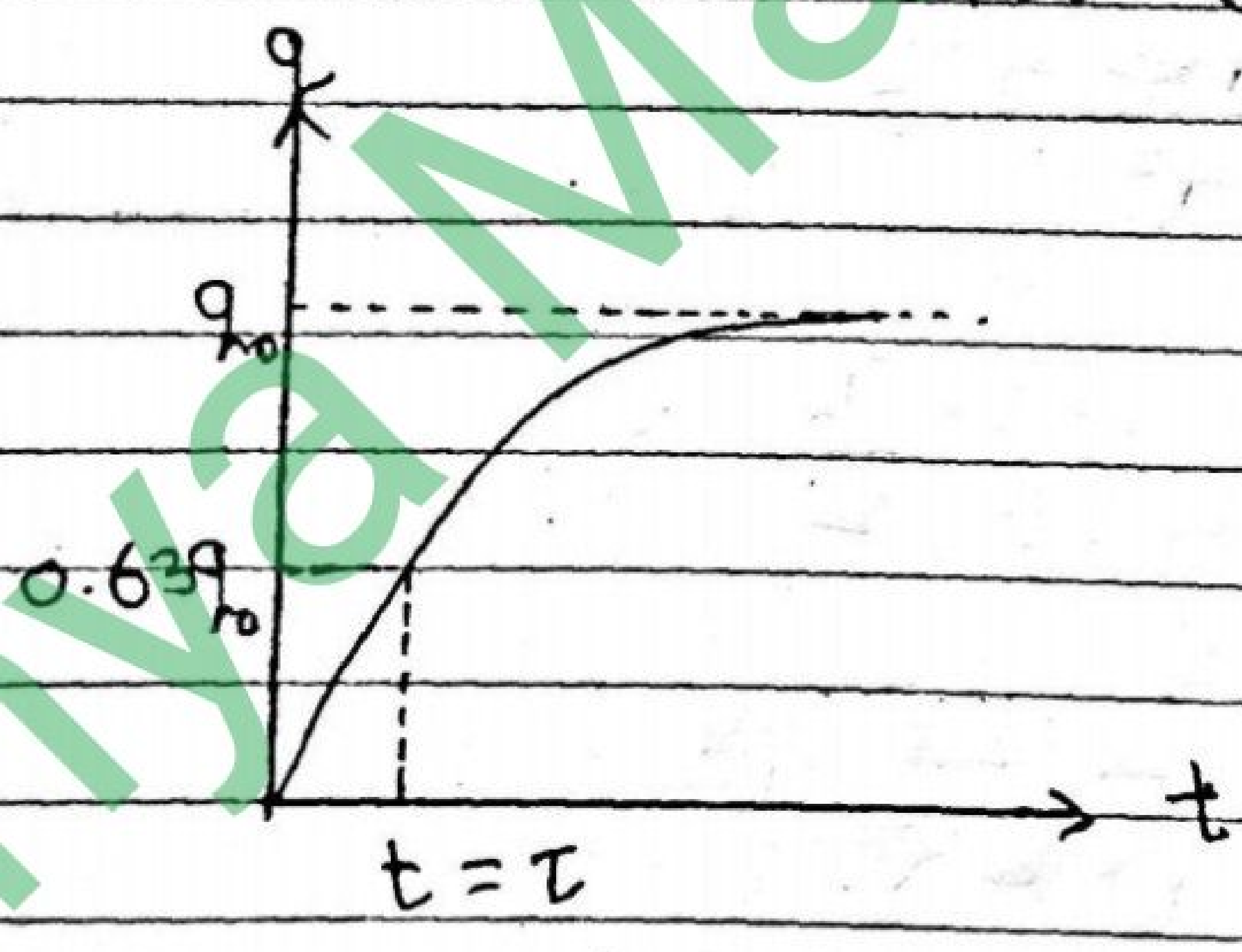
or, $q = CV(1 - e^{-t/CR})$

or, $q = q_0(1 - e^{-t/CR})$ [Where $q_0 = CV$]

or, $q = q_0(1 - e^{-t/\tau})$

Where $\tau = CR$ is charging time constant. This eq^n. explains charging phenomena of the capacitor.

The graphical variation of charging with time is as shown below



Charging time constant

Charging time constant is the time needed at which charge on the capacitor becomes 63% ~~of total charge on the capacito~~ of total charge it can contain

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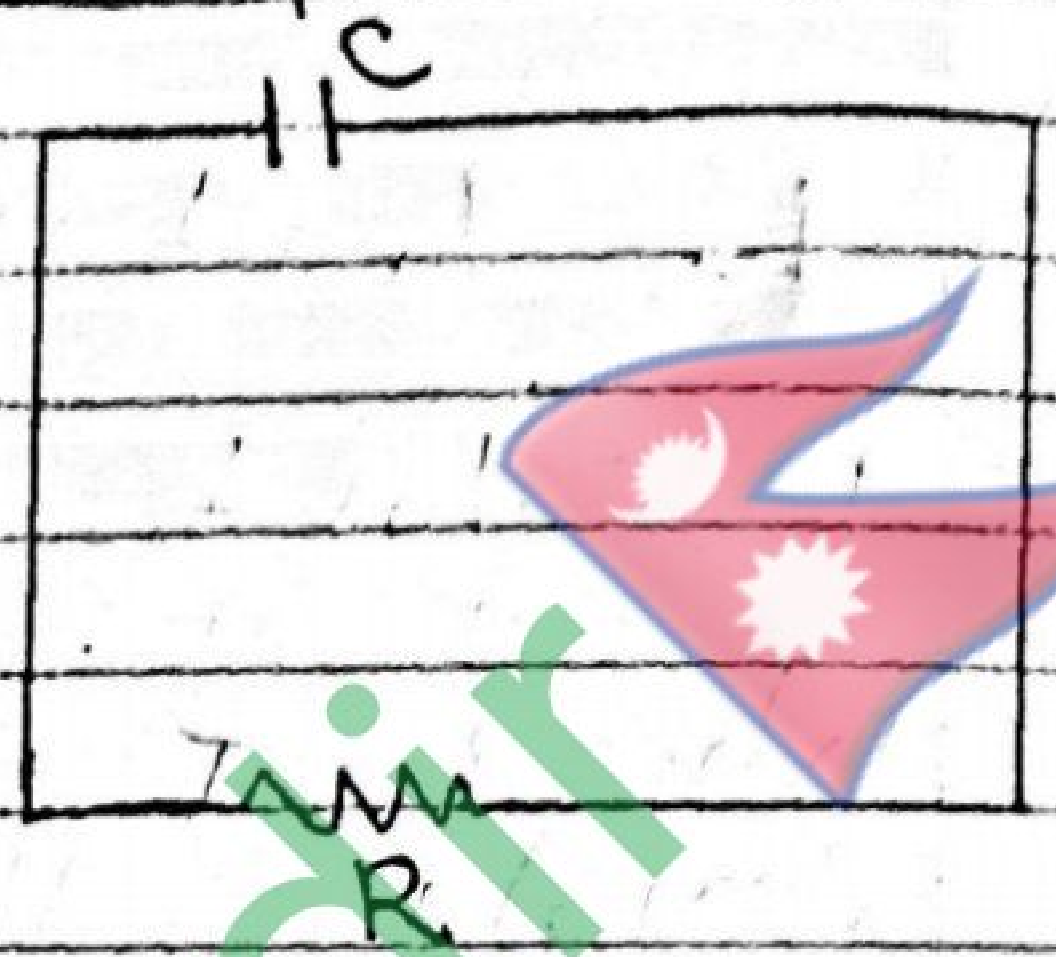
Discharging of the capacitor (Resistor)

Suppose a capacitor of capacitor c is being discharged through a resistor of resistance R as shown in figure.



Here,

$V_c + V_R = 0$

or, $\frac{q}{c} + IR = 0$

or, $\frac{q}{c} + \frac{dq}{dt}R = 0$

or, $\frac{dq}{dt}R = -\frac{q}{c}$

or, $\frac{dq}{q} = -\frac{1}{CR}dt$

On integrating

$\ln q = -\frac{1}{CR}t + K$

or, $q = e^{\left(-\frac{1}{CR}t + K\right)}$

or, $q = e^{-\frac{t}{CR}} \cdot e^{K}$

$$q = Ae^{-\frac{t}{CR}} \longrightarrow [\text{Where } A = e^{k}]$$

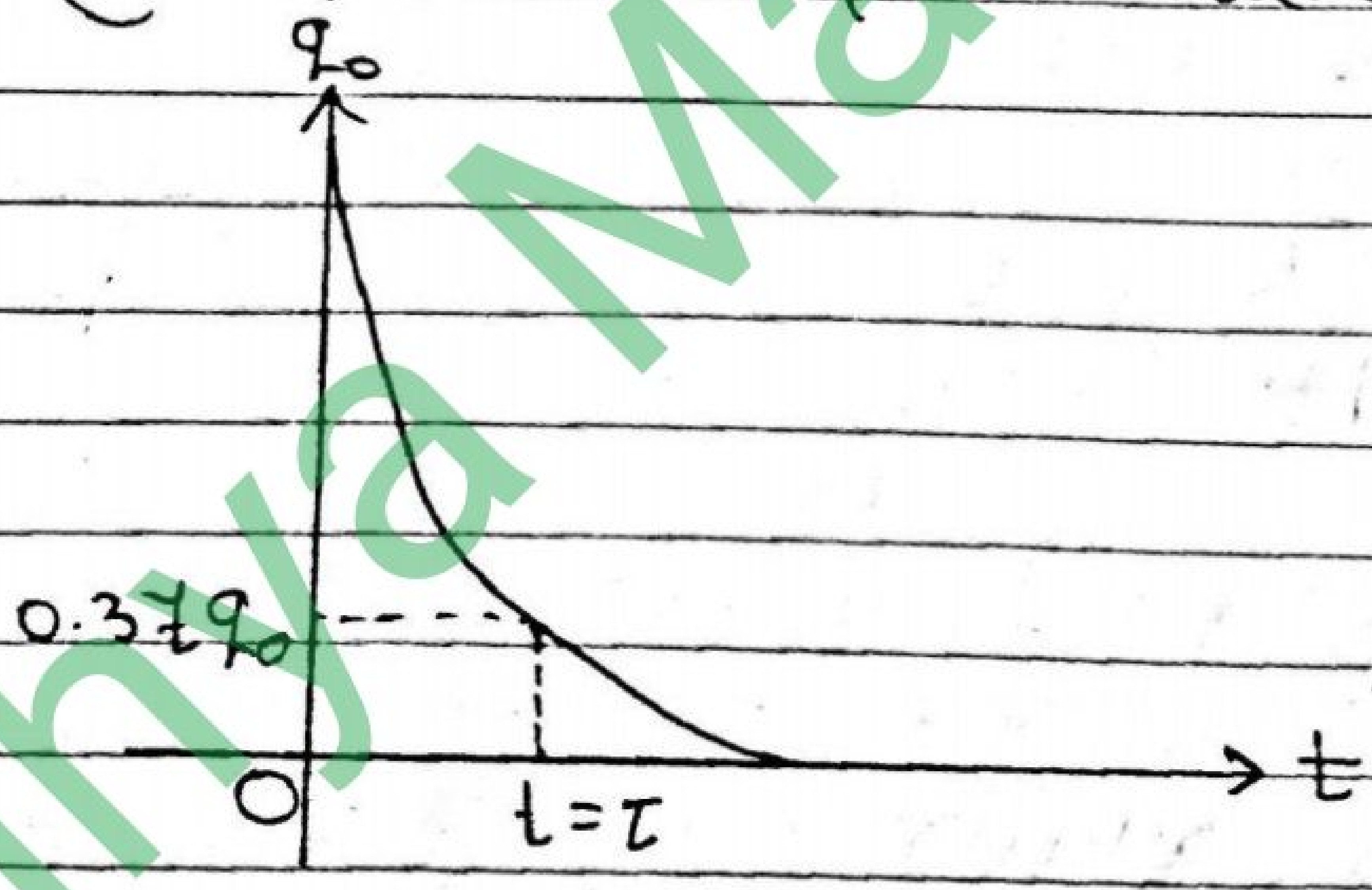
When $t = 0$, $q = q_0$ so, $A = q_0$

Hence,

$$q = q_0 e^{-t/\tau}$$

Here, $\tau = CR$ is discharging time constant. This equation explains the discharging of capacitor in terms of time.

In ghaphical representation,

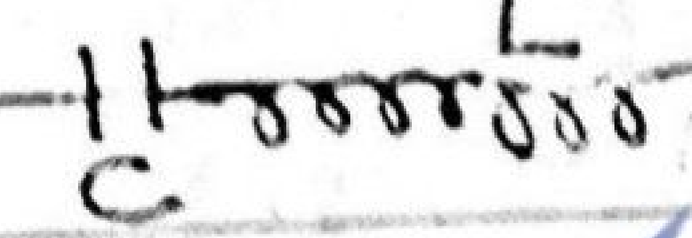


Discharging time constant.

Discharging time constant is the time needed in which the charge remaining on capacitor becomes 37% of total intial charge on it.

Discharging of capacitor through inductor (LC oscillation)

Suppose a capacitor of capacitance 'c' is being discharged through an inductor of inductance 'L' as shown in figure.



$V_L + V_c = 0$

$L\frac{dI}{dt} + \frac{q}{c} = 0$

or, $L\frac{d^2q}{dt^2} + \frac{q}{c} = 0 \quad \left[\because I = \frac{dq}{dt}\right]$

or, $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$

This equation is in the form of $\frac{d^2y}{dt^2} + \omega^2 y = 0$, which is the equation of simple harmonic motion. It means that the discharging of capacitor is oscillating with SHM nature.

On comparing, we get

$\omega^2 = \frac{1}{LC}$

or, $\left(\frac{2\pi}{T}\right)^2 = \frac{1}{LC}$

or, $\frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$

or, $\boxed{T = 2\pi\sqrt{LC}}$

This gives the value of time period of oscillation. And the frequency is

$\therefore \boxed{f = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}}$

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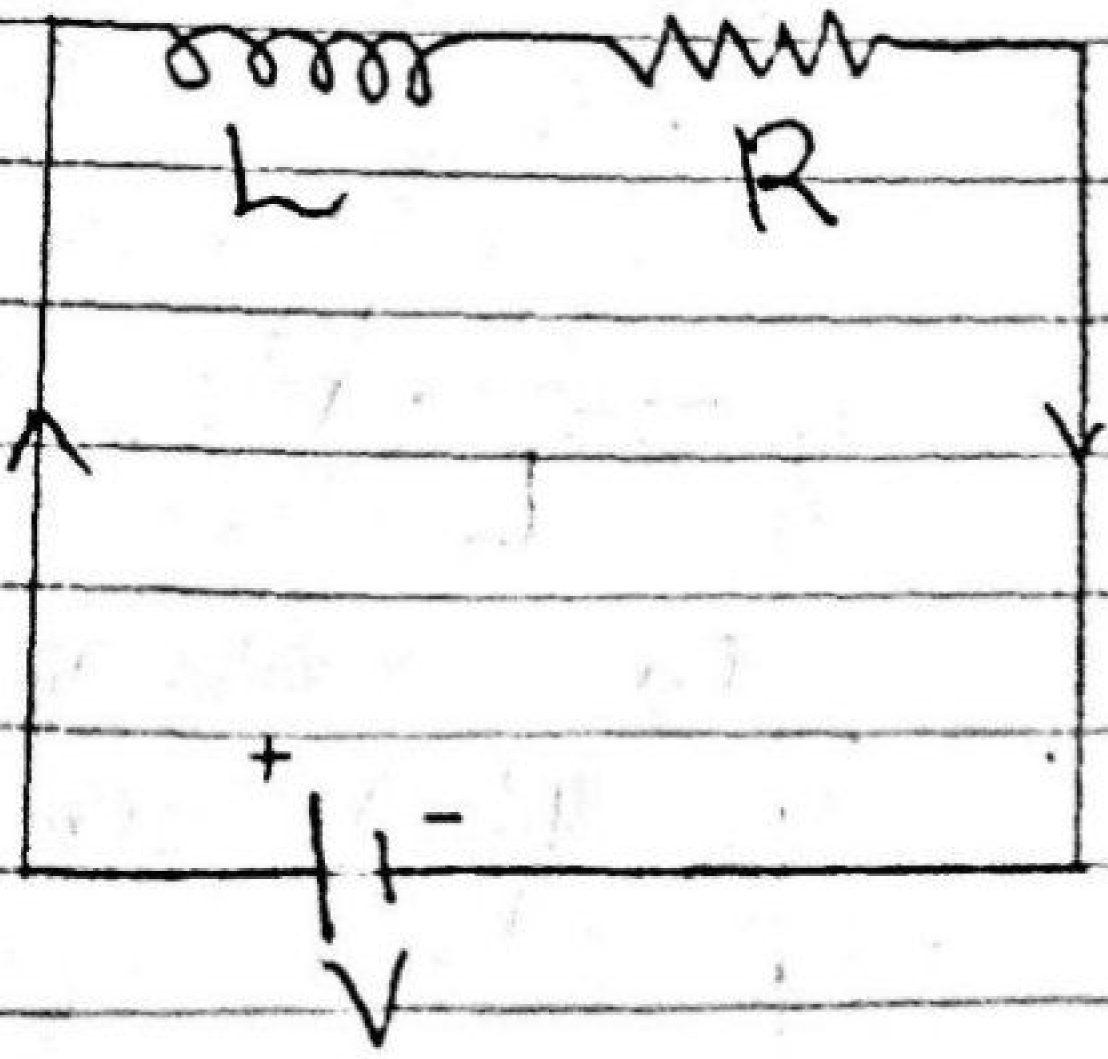
Growing current in an inductor

Suppose an inductor of inductance 'L' is connected in series with resistor of resistance 'R' and is joined to a battery of voltage 'V' as shown in figure.



From figure,

$V = V_R + V_L$

or, $V = IR + L\frac{dI}{dt}$

or, $V - IR = L\frac{dI}{dt} \longrightarrow (1)$

Let $V - IR = x$

$-\frac{dI}{dt}R = \frac{dx}{dt}$

or, $\frac{dI}{dt} = -\frac{1}{R}\frac{dx}{dt}$

Now, equation (1) becomes

$x = L\left(-\frac{1}{R}\frac{dx}{dt}\right)$

or, $-dt \cdot \frac{R}{L} = \frac{dx}{x}$

or, $-\frac{R}{L}dt = \frac{dx}{x}$

On integrating,

$-\frac{R}{L}t + K = \ln x$

or, $\ln x = -\frac{R}{L}t + K$

or, $x = e^{\left(-\frac{R}{L}t + K\right)}$

or, $x = e^{K} \cdot e^{-\frac{R}{L}t}$

or, $x = A e^{-R/L\, t}$ .... [Here, $e^{K} = A$]

or, $V - IR = Ae^{-\frac{R}{L}t}$

At $t=0$, $I=0$

So $V = A$

Now,

$V - IR = Ve^{-\frac{R}{L}t}$

or, $IR = V - Ve^{-R/L\, t}$

or, $IR = V\left(1 - e^{-\frac{R}{L}t}\right)$

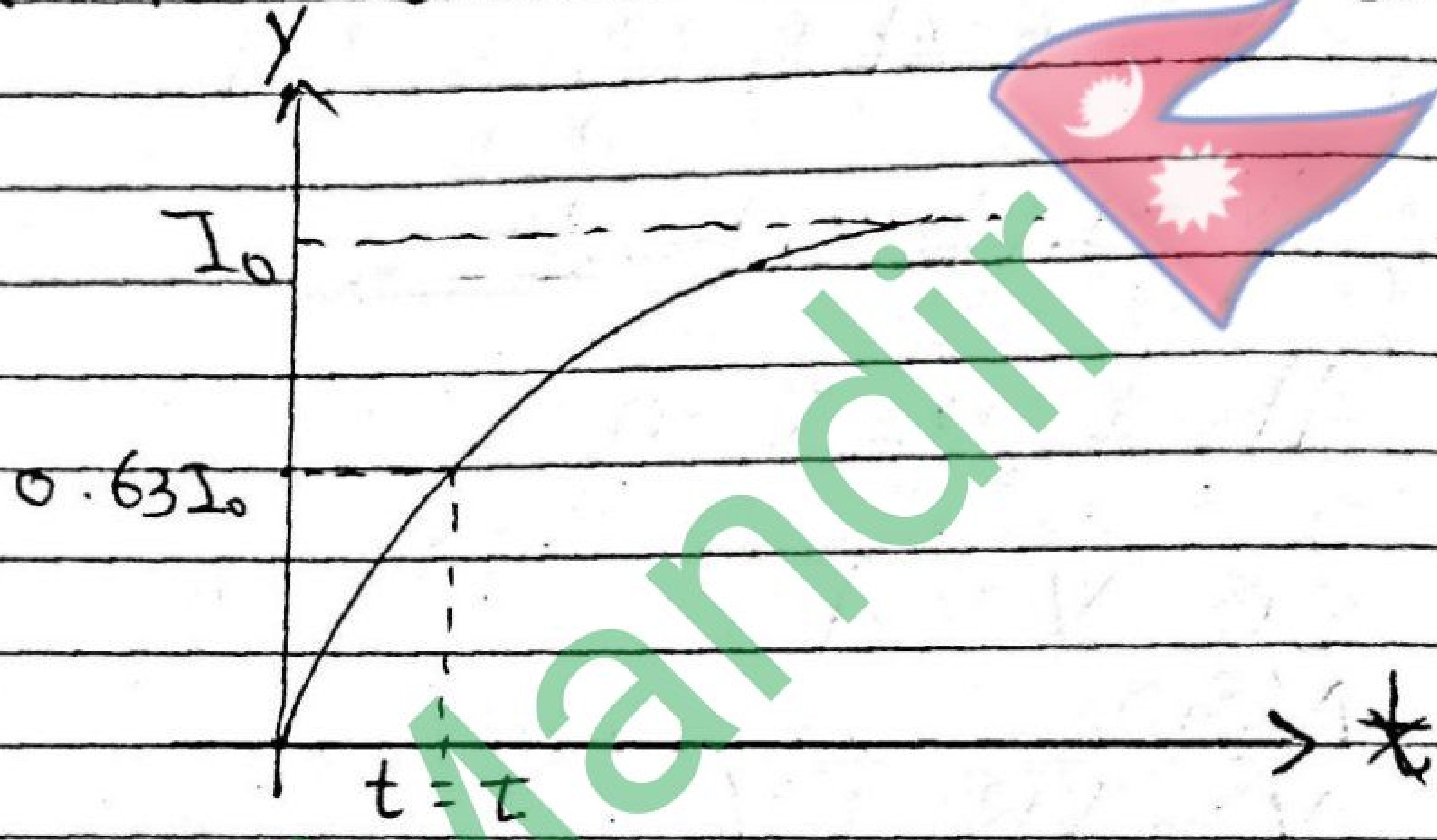
or, $I = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

or, $\boxed{I = I_0\left(1 - e^{-\frac{t}{\tau}}\right)}$

Where, $I_0 = \frac{V}{R}$ is maximum current

$\tau = \frac{L}{R}$ is LR time constant.

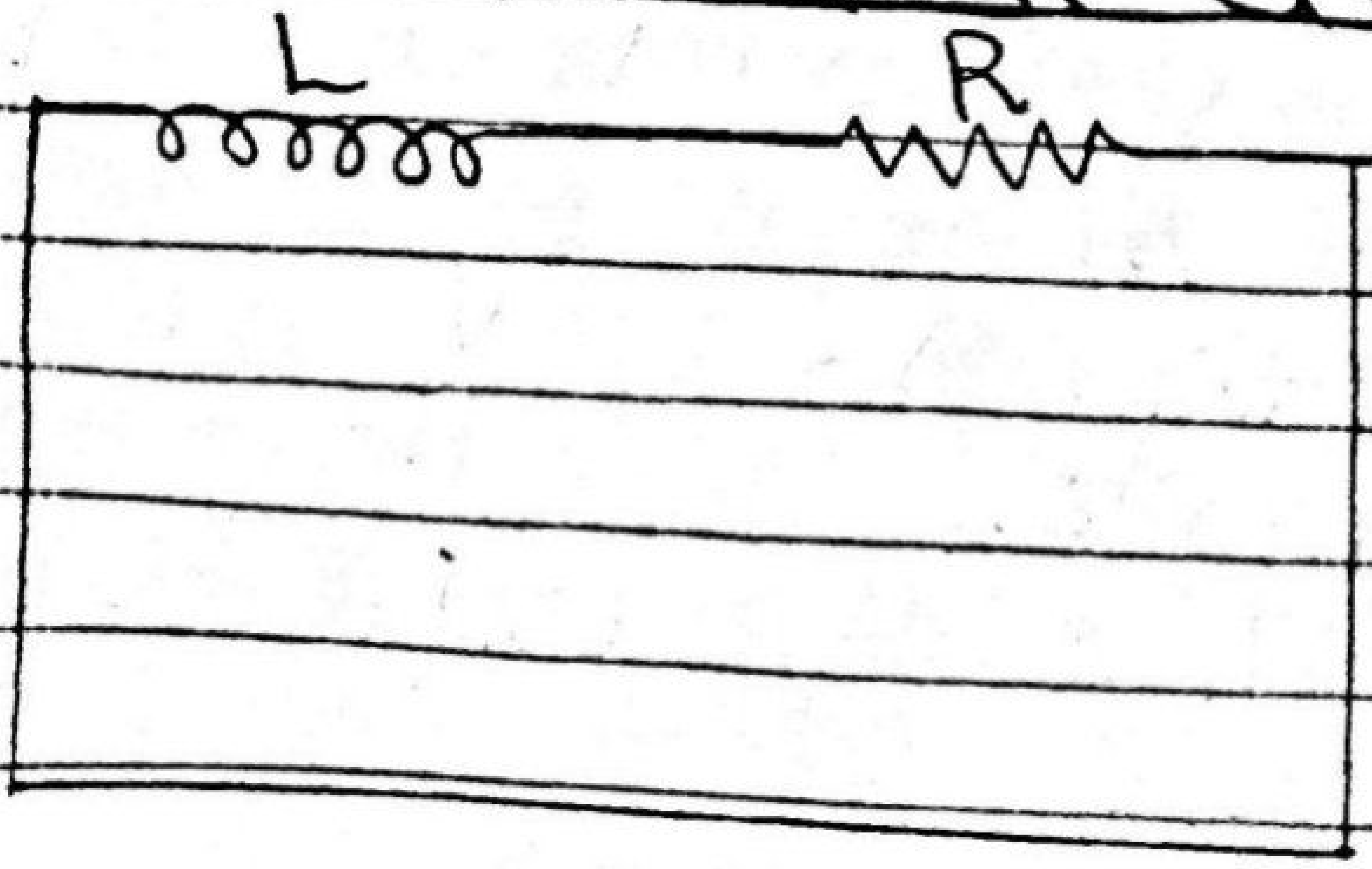
This equation explains the growth of current in LR circuit. The graphical variation of current with time is as shown in fig below.



LR time constant of growing current

LR time constant for growing current in LR circuit becomes 63% of the maximum current in it.

Decay of current in LR circuit



Suppose a fully charge inductor of inductance L is is connected to a resistor of resistance R as shown in figure.

$V_L + V_R = 0$

$L\frac{dI}{dt} + IR = 0$

or, $\frac{dI}{dt} = -\frac{IR}{L}$

or, $\frac{dI}{I} = -\frac{R}{L}dt$

On integrating,

$\ln I = -\frac{R}{L}t + K$

or, $I = e^{\left(-\frac{R}{L}t + K\right)}$

or, $I = e^K \cdot e^{-e/_L t}$

or, $I = Ae^{-R/_L t}$ $[A = e^K]$

At $t = 0$, $I = I_0$. so $A = I_0$

Hence, above equation becomes

$I = I_0 e^{-\frac{R}{L}t}$

Suppose a fully charge inductor of inductance L is is connected to a resistor of resistance R as shown in figure.

$V_L + V_R = 0$

$L\frac{dI}{dt} + IR = 0$

or, $\frac{dI}{dt} = -\frac{IR}{L}$

or, $\frac{dI}{I} = -\frac{R}{L}dt$

On integrating

$\ln I = -\frac{R}{L}t + K$

or, $I = e^{\left(-\frac{R}{L}t + K\right)}$

or, $I = e^{K} \cdot e^{-R/L\,t}$

or, $I = Ae^{-R/L\,t}$ $[A = e^{K}]$

At $t = 0$, $I = I_0$. So $A = I_0$

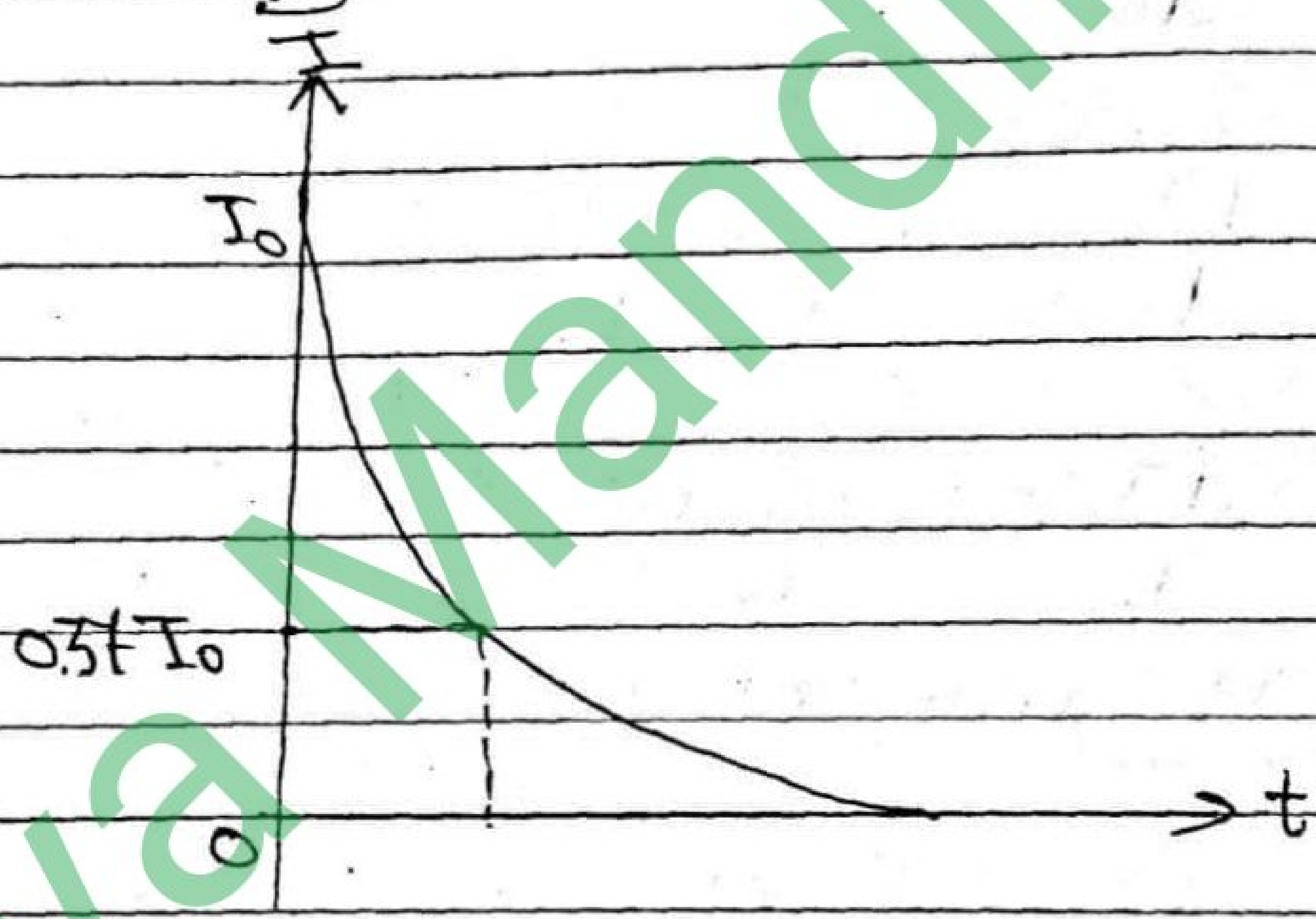
Hence, above equation becomes

$I = I_0 e^{-\frac{R}{L}t}$

$$\boxed{\therefore I = I_0 e^{-\frac{t}{\tau}}}$$

Where, $\tau = \frac{L}{R}$ is the LR time constant for decay current. This eqn. explains the decay of current in the circuit.

The decay is as shown in graph below



LR time constant for decay current

LR time constant for decay current is the time needed at which the current in LR circuit reduces to 37% of intial maximum current.