Unit-1 Co-ordinates & plane

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1. Define plane. Find the equation of plane through the intersection of the planes $2x+3y+10z=8$, $2x-3y+7z=2$ and normal to the plane $3x-2y+4z=5$. [2061, 2071, 2073]

$\Rightarrow$ Solution,

plane:- A surface in which the straight line joining any two points on it lies wholly on the surface is called plane. The general eq^n of plane is $ax+by+cz+d=0$, where a, b, c, d are constant but a, b, c does not all zeros.

$\rightarrow$ Given eq^n of plane.

$2x+3y+10z=8 \longrightarrow (1)$

$2x-3y+7z=2 \longrightarrow (2)$

The eq^n of planes ~~(1) and (2)~~ through the intersection of the given plane (1) and (2)

$2x+3y+10z-8+\lambda(2x-3y+7z-2)=0 \rightarrow (3)$

or, $2x+2\lambda x+3y-3\lambda y+10z+7\lambda z-8-2\lambda=0$

or, $(2+2\lambda)x+(3-3\lambda)y+(10+7\lambda)z-(8+2\lambda)=0$

If this plane normal to the plane $3x-2y+4z=5$ then

$(2+2\lambda)\cdot 3+(3-3\lambda)\cdot(-2)+(10+7\lambda)\cdot 4=0$

or, $40\lambda=-40$

$\therefore \lambda=-1$

Using this value of λ in eq^n (3), we get

~~$2y+z=2$~~ $2y+z-2=0$.

2 Find the equation of the plane through (α, β, γ) parallel to $ax+by+cz+d=0$ [2062, 2073]

⇒ Solution,

Given plane is

$ax+by+cz+d=0 \longrightarrow (1)$

Any plane parallel to the plane (1) is

$ax+by+cz+K=0 \longrightarrow (2)$

where K is constant.

Since plane (2) passes through (α, β, γ)

$a\alpha+b\beta+c\gamma+K=0$

or, $K=-(a\alpha+b\beta+c\gamma)$

Using this value of K in eqn (2)

$ax-a\alpha+by-b\beta+cz-c\gamma=0$

or, $a(x-\alpha)+b(y-\beta)+c(z-\gamma)=0$.

Which is required eqn of plane

3 What is the relation between the direction cosines of the two lines normal to two perpendicular planes? Find the eqn of the plane which is perpendicular to the plane $5x+3y+6z+8=0$ and which contains the line of intersection of the planes $x+2y+3z-4=0$, $2x+y-z+5=0$ [2054]

⇒ Solution,

Let the given plane be

$a_1x+b_1y+c_1z+d_1=0 \text{ ------- } (1)$

$a_2x+b_2y+c_2z+d_2=0 \text{ ------- } (2)$

The d.c.s of the line normal to plane (1) and (2) are proportional to a_1, b_1, c_1 and a_2, b_2 and c_2 respectively. So

$$l_1 = \frac{a_1}{\sqrt{a_1^2+b_1^2+c_1^2}}, \quad m_1 = \frac{b_1}{\sqrt{a_1^2+b_1^2+c_1^2}}$$

$$n_1 = \frac{c_1}{\sqrt{a_1^2+b_1^2+c_1^2}} \;\&\; l_2 = \frac{a_2}{\sqrt{a_2^2+b_2^2+c_2^2}},$$

$$m_1 = \frac{b_2}{\sqrt{a_2^2+b_2^2+c_2^2}}, \quad n_1 = \frac{c_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

respectively. Since these two lines are perpendicular

$\therefore l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

i.e. $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$.

Next part :-

Any plane through the intersection of planes

$x+2y+3z-4=0$ and $2x+y+z+5=0$ is

$$x+2y+3z-4+\lambda(2x+y-z+5)=0 \longrightarrow (1)$$

or, $(1+2\lambda)x+(2+\lambda)y+(3-\lambda)z-(4-5\lambda)=0 \longrightarrow (2)$

The direction cosines of the line normal to plane (1) are proportional to

$(1+2\lambda), (2+\lambda), (3-\lambda)$

Also the given plane is..

$5x+3y+6z+8=0 \longrightarrow (2)$

The d.c.s of any line normal to plane (2) are proportional to 5, 3, 6.

Since plane (1) and (2) are perpendicular to each other, so, using perpendicularity condition, we get

$(1+2\lambda)5+(2+\lambda)3+(3-\lambda)6=0$

$7\lambda+29=0$

$\lambda=-\frac{29}{7}$

Using the value of λ in eq^n (1)

$x+2y+3z-4-\frac{29}{7}(2x-y-z+5)=0$

Or, $51x+15y-50z+173=0$

Hence, which is required eq^n of plane.

4. Define angle between two planes. Obtain the eq^n of plane which passes through the point (-1, 3, 2) and is perpendicular to each of two planes; $x+2y+2z=5$, $3x+3y+2z=8$. [2054]

$\Rightarrow$ The angle between two planes is defined as the angle between normal to these plane.

Next part

$$\begin{array}{c|cccc} 1 & 2 & 2 & 1 & 2 \\ 3 & 3 & 2 & 3 & 3 \end{array}$$

cross multiplication rule

Equation of any plane through point $(-1,3,2)$ is
$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$
$a(1+x)+b(y-3)+c(z-2)=0 \longrightarrow (1)$
Where a,b,c are d.r.s. of the line normal to the plane. If the plane (1) perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$ then using perpendicular condition
$a+2b+2c=0 \longrightarrow (2)$
$3a+3b+2c=0 \longrightarrow (3)$
Solving eq^n (2) & (3), we get

$$\frac{a}{2\times2-2\times3}=\frac{b}{2\times3-1\times2}=\frac{c}{1\times3-2\times3}$$

or, $\frac{a}{-2}=\frac{b}{4}=\frac{c}{-3}$

or, $\frac{a}{2}=\frac{b}{-4}=\frac{c}{-3}$ (k say)

$a=2k, b=-4k, c=3k$
Using these values of a,b,c in eq^n (1), we get
$2k(1+x)+(-4k)(y-3)+3k(z-2)=0$
or, $2+2x+12-4y+3z-6=0$
or, $2x-4y+3z+8=0$

Which is required eq^n of plane.

5. Show that the straight lines whose direction cosines are given by the relation $al+bm+cn=0$ and $fmn+gnl+hlm=0$ are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$. [2054]

⇒ Solution,
Let the given relations of the d.c.s of the straight lines are

$al+bm+cn=0 \longrightarrow (1)$

$fmn+gnl+hlm=0 \longrightarrow (2)$

From equation (1),

$n = -\frac{al+bm}{c}$

Eliminating n from equation (1) and (2), we get,

$$fm\left(-\frac{al+bm}{c}\right)+g\left(-\frac{al+bm}{c}\right)l+hlm=0$$

or, $-fmal-fm^2b-agl^2-bgml+hlmc=0$

or, $lm(af+bg-ch)+agl^2+fm^2b=0$

or, $ag\left(\frac{l}{m}\right)^2+(af+bg-ch)\frac{l}{m}+bf=0$

Which is quadric equation in $\frac{l}{m}$, so, It has two roots $\frac{l_1}{m_1}, \frac{l_2}{m_2}$ say.

$\therefore$ product of roots, $\frac{l_1}{m_1} \cdot \frac{l_2}{m_2} = \frac{bf}{ag}$

or, $\frac{l_1 l_2}{f/a} = \frac{m_1 m_2}{g/b} \longrightarrow (3)$

Similarly,

$\frac{l_1 l_2}{f/a} = \frac{n_1 n_2}{h/c} \longrightarrow (4)$

combining eq^n (3) and (4), we get,

$$\frac{l_1 l_2}{f/a} = \frac{m_1 m_2}{g/b} = \frac{n_1 n_2}{h/c}$$

The given lines will be perp. to each other, we have;

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

Hence, proved

6. Show that the straight lines whose d.c.s are given by the relations $ul + vm + wn = 0$ and $al^2 + bm^2 + cn^2 = 0$ are perpendicular to $u^2(b+c) + v^2(c+a) + w^2(a+b) = 0$.

$\Rightarrow$ Solution,

Given relation of d.c.s of straight lines are

$ul + vm + wn = 0 \longrightarrow (1)$

$al^2 + bm^2 + cn^2 = 0 \longrightarrow (2)$

From eqn. (1),

$n = -\left(\frac{ul + vm}{w}\right)$

Eliminating n from eqn. (1) and (2)

$al^2 + bm^2 + c\left(\frac{(ul+vm)^2}{w^2}\right) = 0$

or, $al^2 + bm^2 + c\left(\frac{u^2l^2 + v^2m^2 + 2ulvm}{w^2}\right) = 0$

or, $al^2w^2 + bm^2w^2 + cu^2l^2 + cv^2m^2 + 2culvm = 0$

or, $l^2(aw^2 + cu^2) + m^2(bw^2 + cm^2) + 2culvm = 0$

or, $(aw^2 + cu^2)\left(\frac{l}{m}\right)^2 + (bw^2 + cm^2) + 2cuv\left(\frac{l}{m}\right) = 0$

or, $(aw^2 + cu^2)\left(\frac{l}{m}\right)^2 + 2cuv\frac{l}{m} + (bw^2 + cv^2) = 0$

which is quadratic in $\frac{l}{m}$. It has two roots $\frac{l_1}{m_1}, \frac{l_2}{m_2}$ (say).

~~The lines will be perp. if~~

~~$l_1l_2 + m_1m_2 + n_1n_2 = 0$ then,~~ $\longrightarrow (3)$

Product of two roots, $\frac{l_1}{m_1} \cdot \frac{l_2}{m_2} = \frac{bw^2+cv^2}{aw^2+cu^2}$

~~Similarly~~, or, $\frac{l_1 l_2}{bw^2+cv^2} = \frac{m_1 m_2}{aw^2+cu^2} \longrightarrow (3)$

Similarly,

$\frac{l_1 l_2}{bw^2+cv^2} = \frac{n_1 n_2}{av^2+bu^2} \longrightarrow (4)$

Combing (3) and (4), we get

$\frac{l_1 l_2}{bw^2+cv^2} = \frac{m_1 m_2}{aw^2+cu^2} = \frac{n_1 n_2}{av^2+bu^2}$

If given two lines ~~are~~ will be perp. to each other if

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

or, $bw^2+cv^2+aw^2+cu^2+av^2+bu^2 = 0$

or, $u^2(b+c)+v^2(c+a)+w^2(a+b) = 0$

Hence proved

7. Find the equation of plane through the points $(-1, 1, -1)$ and $(6, 2, 1)$ normal to the plane $2x+y+z=5$. [2059, 2068, 2070, 2071]

⇒ Solution,
Equation of the plane through $(-1, 1, -1)$ is
$a(x+1)+b(y-1)+c(z+1)=0 \longrightarrow (1)$
Since, the plane (1) passing through $(6,2,1)$

$a(6+1)+b(2-1)+c(1+1)=0$
or, $7a+b+2c=0 \longrightarrow (2)$
According to the questions, plane (1) is normal to the plane $2x+y+z=5$.
So using perpendicularity condition,
$2a+b+c=0 \longrightarrow (3)$
Solving eq^n. (2) and (3) by cross-multiplication rule,

$$\begin{array}{cccccc} & a & & b & & c \\ 7 & 1 & 2 & 7 & 1 \\ 2 & 1 & 1 & 2 & 1 \end{array}$$

$$\frac{a}{1-2} = \frac{b}{4-7} = \frac{c}{7-2}$$

or, $\dfrac{a}{-1} = \dfrac{b}{-3} = \dfrac{c}{5}$

or, $\dfrac{a}{1} = \dfrac{b}{3} = \dfrac{c}{-5} = k \text{(say)}$

which gives us,
$a = k, b = 3k, c = -5k$
Substituting the value of a, b, c in eqn (1)
we get,
$k(x+1) + 3k(y-1) - 5k(z+1) = 0$
or, $x + 1 + 3y - 3 - 5z - 5 = 0$
or, $x + 3y - 5z - 7 = 0$
Which is required eqn of plane.

8. Find the equation of plane through (2, -3, 1) normal to the line joining (3, 4, -1) and (2, -1, 5). [2060, 2067, 2068, 2070]

⇒ Solution,
The eqn of plane through (2, -3, 1) is
$a(x-2) + b(y+3) + c(z-1) = 0 \longrightarrow (1)$
Where a, b, c are direction ratios of the normal of plane (1). The d.c.s of line joining the points (3, 4, -1) and (2, -1, 5) ~~is~~ are proportional to
$2-3, -1-4, 5+1$
i.e. $-1, -5, 6$
Since, plane (1) is perp. to the line joining (3, 4, -1) and (2, -1, 5), so d.r.s must be proportional.
i.e. $\frac{a}{-1} = \frac{b}{-5} = \frac{c}{6} = k$ (say)
$\therefore a = -k, b = -5k, c = 6k$

Substituting the value of a, b, c in eqn. (1) we get

$-1k(x-2) - 5k(y+3) + 6k(z-1) = 0$

or, $-x + 2 - 5y - 15 + 6z - 6 = 0$

or, $x + 5y - 6z + 19 = 0$
which is the required eqn. of the plane.

9. Show that the plane $x + 2y - 3z = -4$ is perpendicular to each of the planes $2x + 5y + 4z + 1 = 0$ and $4x + 7y + 6z + 2 = 0$. [BA/BSC 2064]

⇒ Solution,
Equation of given plane is
$x + 2y - 3z + 4 = 0 \longrightarrow (1)$
The d.c.s. of the line normal to plane (1) are proportional to 1, 2, -3.
Also, the given planes are
$2x + 5y + 4z + 1 = 0 \longrightarrow (2)$
$4x + 7y + 6z + 2 = 0 \longrightarrow (3)$
The d.c.s of the line normal to the plane (2) are proportional to 2, 5, 4.
Since, eqn. (1) and (2) are perp. to each other
$1 \times 2 + 2 \times 5 + (-3) \times 4 = 0$
or, $2 + 10 - 12 = 0$
or, $0 = 0$ (True)

Hence, plane (1) is perp. to plane (2).
Also,
The d.c.s of the line normal to plane (3) are proportional to 4, 7, 6.
Since, plane (1) and (3) are perp. to each other, then,

$1\times4 + 2\times7 - 3\times6 = 0$

or, $4 + 14 - 18 = 0$

$0 = 0$ (True)

Hence, plane (1) is perpendicular to plane (3).

10. Obtain the equation of the plane through the intersection of the planes $x+2y+3z+4=0$ and $4x+3y+2z+1=0$ and the origin. [2054, 2070]

→ Solution,
The eq^n of the plane through the intersection of the planes $x+2y+3z+4=0$ and $4x+3y+2z+1=0$ is

$(x+2y+3z+4)+\lambda(4x+3y+2z+1) = 0 \longrightarrow (1)$

Since, the eq^n (1) passes through the origin

so, $(0+0+0+4)+\lambda(0+0+0+1)=0$

$\therefore \lambda = -4$

Substituting value of λ in eq^n (1), we get

$x+2y+3z+4-4(4x+3y+2z+1)=0$

or, $x+2y+3z+4-16x-12y-8z-4=0$

or, $-15x-10y-5z=0$

$\therefore 3x+2y+z=0$

Which is required eq^n of plane.

11. Find the equation of the plane through the intersection of the planes $ax+by+cz+d=0$, $a_1x+b_1y+c_1z+d_1=0$ and perp. to the xy plane [2058]

$\Rightarrow$ Solution,

The eq^n of the plane through the intersection of the planes $ax+by+cz+d=0$, $a_1x+b_1y+c_1z+d_1=0$ is

$$(ax+by+cz+d)+\lambda(a_1x+b_1y+c_1z+d_1)=0 \rightarrow (1)$$

or, $ax+by+cz+d+\lambda a_1x+\lambda b_1y+\lambda c_1z+\lambda d_1=0$

or, $(a+\lambda a_1)x+(b+\lambda b_1)y+(c+\lambda c_1)z+(d+\lambda d_1)=0 \rightarrow (2)$

So, d.r.s of the line normal to the plane (2) are proportional to $a+\lambda a_1$, $b+\lambda b_1$, $c+\lambda c_1$

Also, d.r.s of the line normal to xy-plane i.e. z-axis are 0, 0, 1.

Using perpendicularity condition.

$(a+\lambda a_1)0+(b+\lambda b_1)0+(c+\lambda c_1)1=0$

$\therefore \lambda = -\dfrac{c}{c_1}$

Substituting the value of λ in eq^n (1),

$ax+by+cz+d-\frac{c}{c_1}(a_1x+b_1y+c_1z+d_1)=0$

or, $ac_1x+bc_1y+cc_1z+c_1d-ca_1x-cb_1y-cc_1z+cd_1=0$

or, $(ac_1-ca_1)x+(bc_1-cb_1)y+(cc_1-cc_1)z+(dc_1-cd_1)=0$

$\therefore (ac_1-a_1c)x+(bc_1-b_1c)y+(dc_1-d_1c)=0$

Which is required eqn. of plane.

12. Obtain the angle between the two planes represented by $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ [2070]

⇒ Solution,

Given eqn. is

$ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0 \longrightarrow (1)$

Let two planes represented by (i) is

$l_1x+m_1y+n_1z=0$ and $l_2x+m_2y+n_2z=0$

Where l_1, m_1, n_1 and l_2, m_2, n_2 are d.c.s of lines normal to those of respective planes.

So, we have

$(l_1x+m_1y+n_1z)(l_2x+m_2y+n_2z)=ax^2+by^2+cz^2+2fyz+2gzx+2hxy$ comparing the coeff. of like term

$a=l_1l_2$, $b=m_1m_2$, $c=n_1n_2$, $2f=m_1n_2+m_2n_1$, $2g=n_1l_2+l_1n_2$, $2h=l_1m_2+l_2m_1$

If θ be the angle betn. the planes (2)

$$ax+by+cz+d-\frac{c}{c_1}(a_1x+b_1y+c_1z+d_1)=0$$

or, $ac_1x+bc_1y+cc_1z+c_1d-ca_1x-cb_1y-cc_1z+cd_1=0$

or, $(ac_1-ca_1)x+(bc_1-cb_1)y+(cc_1-cc_1)z+(dc_1-d_1c)=0$

$\therefore (ac_1-a_1c)x+(bc_1-b_1c)y+(dc_1-d_1c)=0$

Which is required eq^n of plane.

12. Obtain the angle between the two planes represented by $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ [2070]

$\Rightarrow$ Solution,

Given eq^n is

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0 \longrightarrow (1)$$

Let two planes represented by (i) is

$l_1x+m_1y+n_1z=0$ and $l_2x+m_2y+n_2z=0$

Where l_1, m_1, n_1 and l_2, m_2, n_2 are d.c.s of lines normal to those of respective planes.

So, we have

$(l_1x+m_1y+n_1z)(l_2x+m_2y+n_2z)=ax^2+by^2+cz^2+2fyz+2gzx+2hxy$ comparing the coeff. of like term

$a=l_1l_2$, $b=m_1m_2$, $c=n_1n_2$, $2f=m_1n_2+m_2n_1$, $2g=n_1l_2+l_1n_2$, $2h=l_1m_2+l_2m_1$

If θ be the angle bet^n the planes (2) then,

$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$ and

$\sin\theta = \sqrt{\Sigma(l_1m_2 - l_2m_1)^2}$

So, $\tan\theta = \dfrac{\sqrt{\Sigma(l_1m_2 - l_2m_1)^2}}{l_1l_2 + m_1m_2 + n_1n_2}$

$= \dfrac{\sqrt{(l_1m_2+l_2m_1)^2 - 4l_1l_2m_1m_2 + (m_1n_2+n_1m_2)^2 - 4m_1n_1m_2n_2 + (n_1l_2+n_2l_1)^2 - 4n_1l_1n_2l_2}}{l_1l_2 + m_1m_2 + n_1n_2}$

$= \dfrac{\sqrt{4h^2 - 4ab + 4f^2 - 4bc - 4g^2 - 4ac}}{a+b+c}$

$= \dfrac{2\sqrt{f^2+g^2+h^2-ab-bc-ca}}{a+b+c}$

$\therefore \theta = \tan^{-1}\left(\dfrac{2\sqrt{f^2+g^2+h^2-ab-bc-ca}}{a+b+c}\right)$

13. Prove that the equation $6x^2+4y^2-10z^2+3yz+4zx-11xy=0$ represents a pair of perpendicular planes. [2056]

⇒ Solution,

Given eqn. is

$6x^2+4y^2-10z^2+3yz+4zx-11xy=0 \longrightarrow (1)$

Comparing eqn. (1) with homogeneous eqn. of second degree

$ax^2+by^2+cz^2+2hxy+2fyz+2gzx=0$

we get
$a=6,\ b=4,\ c=-10,\ f=\frac{3}{2},\ g=2,\ h=-\frac{11}{2}$

The eqn (1) represents perp. planes if
$abc+2fgh-af^2-bg^2-ch^2=0$ and $a+b+c=0$
For this
$abc+2fgh-af^2-bg^2-ch^2$
$=6\times4\times(-10)+2\times\frac{3}{2}\times2\times\left(-\frac{11}{2}\right)-6\left(\frac{3}{2}\right)^2-4(2)^2-$

$\left\{(-10)\times\left(-\frac{11}{2}\right)^2\right\}$

$=-240-33-\frac{27}{2}-16+\frac{605}{2}=0$

$=-289+289=0$
Again, $a+b+c=6+4-10=0$
Hence given eqn (1) represents pair of perpendicular plane.

14. Find the equation of the plane containing the lines through the origin with direction cosines proportional to $(1,-2,+2)$ and $(5,2,-3)$.
⇒ Solution,
The eqn of plane through the origin is
$ax+by+cz=0 \longrightarrow (1)$
The d.c.s of plane (1) is proportional to a, b, c
Also, given d.c.s. proportional to $(1,-2,+2)$

(5, 2, -3).

Using perpendicularity condition,

So, $a \cdot 1 + b \cdot (-2) + c \cdot 2 = 0 \longrightarrow (2)$

$a \cdot 5 + b \cdot 2 + c \cdot (-3) = 0 \longrightarrow (3)$

Solving eqn (2) and (3) by cross ~~mult~~ multiplication rule,

$$\frac{a}{2} = \frac{b}{7} = \frac{c}{12} = K \text{ (say)}$$

which gives

$a = 2K,\ b = 7K,\ c = 12K$

Substituting the values of a, b, c in eqn (1), we get

$2Kx + 7Ky + 12Kz = 0$

$2x + 7y + 12z = 0$

which is required eqn of plane.

15. Find the angle between the ~~pla~~ pair of plane $x + 2y + 3z = 4$, $3x + 4y + 5z = 10$

⇒ Solution,

The angle between two planes : $x + 2y + 3z = 4$ and $3x + 4y + 5z = 10$ is θ as given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{1.3 + 2.4 + 3.5}{\sqrt{1+4+9}\sqrt{9+16+25}}$$

$$= \frac{3+8+15}{\sqrt{14}\sqrt{50}}$$

$$= \frac{26}{10\sqrt{7}}$$

$$= \frac{13}{5\sqrt{7}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{13}{5\sqrt{7}}\right) \text{ Ans}$$

16. For what value of k will the two plane $2x+3y+4z=7$ and $2x-ky+5z=9$ is perpendicular?

⇒ Solution,

Given planes are $2x+3y+4z=7$ and $2x-ky+5z=9$

If they are perp. to each other, Then

$2\cdot 2 + 3\cdot(-k) + 4\cdot 5 = 0$

or, $4 - 3k + 20 = 0$

$\therefore k = 8.$

Hence, the value of k is 8.

17 Show that $ax+by+d=0$ represents a plane parallel to the z-axis. Find the equation of the plane through the points (1,2,3) and (1,5,6) parallel to the x-axis.

⇒ Solution,

Given plane is

$ax+by+d=0 \longrightarrow (1)$

Here, the d.r.s of normal of plane are (a,b,0). We know that the d.c.s of z-axis are (0,0,1).

If the plane is parallel to z-axis then its normal must be perp. to z-axis.

$a.0+b.0+0.1=0$

$0=0$

which is true

Hence, the plane is parallel to z-axis.

2nd part Again

We know that,

The eqn of plane parallel to x-axis is

$by+cz+d=0 \longrightarrow (2)$

plane (2) is passes through the points (1,2,3) and (1,5,6), then

$2b+3c+d=0$ and

$5b+6c+d=0$

solving this, we get

$$\frac{a}{0}=\frac{b}{1}=\frac{c}{-1}=\frac{d}{1}=k \text{ (say)}$$

Which gives,

$b = K,\ c = -K,\ d = K$

substituting the value of b, c, d in eqn.(2)

$Ky + (-K)z + K = 0$

$y - z + 1 = 0$

Which is required eqn. of plane.

18. Find the equation of the plane through the three points $(1,1,0)$, $(1,2,1)$ and $(-2,2,-1)$.

⇒ solution,

The eqn. of the plane pass through $(1,1,0)$ is $a(x-1) + b(y-1) + cz = 0 \longrightarrow (1)$

plane (1) is passes through $(1,2,1)$ and $(-2,2,-1)$

Then,

$a(1-1) + b(2-1) + c \cdot 1 = 0$

$b + c = 0$ and $\longrightarrow (2)$

$a(-2-1) + b(2-1) + c(-1) = 0$

$-3a + b - c = 0 \longrightarrow (3)$

Solving eqn.(2) and (3), we get,

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{-3} = K \text{ (say)}$$

Which gives,

$a = 2K,\ b = 3K,\ c = -3K$

Substituting the value of a, b, c in eqn.(1)

$2K(x-1) + 3K(y-1) + (-3K)z = 0$

$2x + 3y - 3z - 5 = 0$

which is required eqn. of plane.

19 Show that the four points (1, 3, -1), (1, 1, 0), (2, 5, 4) and (2, 1, 3) are coplanar

⇒ Solution,

Let the eqn. of plane is

$ax+by+cz+d=0 \longrightarrow (1)$

plane (1) passes through the first three points (1, 3, -1) (1, 1, 0) (2, 5, 4),

so, $a+3b-1c+d=0 \longrightarrow (2)$

$a+b+d=0 \longrightarrow (3)$

$2a+5b+4c+d=0 \longrightarrow (4)$

Substracting (2) from (1), we get

$a(x-1)+b(y-3)+c(z+1)=0 \longrightarrow (5)$

Again, substracting eqn. (3) from (2) and (4) from (2), we get

$2b-c=0$ and

$a+2b+5c=0$

Solving this by using cross multiplication rule,

$\frac{a}{12} = \frac{b}{-1} = \frac{c}{-2} = k$ (say)

which gives us,

$a=12k, \; b=-k, \; c=-2k$

substituting the values of a, b, c in eqn. (5), we get

$12k(x-1)-k(y-3)-2k(z+1)=0$

or, $12x-y-2z-11=0$

The given four points will be coplanar

if last one point (2, 7, 3) also lies in the same plane

i.e. $12 \times 2 - 7 - 2 \times 3 - 11 = 0$

or, $24 - 7 - 6 - 11 = 0$

or, $0 = 0$

Which is true.

Hence, The given four points are coplanar.

20 Obtain the equation of the plane normal to the yz-plane and passing through the points (1, -2, 4) and (3, -4, 5)

⇒ Solution,

The eqn of the plane normal to yz-plane (x-axis), then the d.c.s of yz-plane is proportional to (1, 0, 0)

Also,

The eqn of the plane passing through the point (1, -2, 4) is

$a(x-1) + b(y+2) + c(z-4) = 0 \longrightarrow (1)$

plane (1) passes through (3, -4, 5) is

$a(3-1) + b(-4+2) + c(5-4) = 0$

or, $2a - 2b + c = 0 \longrightarrow (2)$

Using cross multiplication rule, we get

$$\frac{a}{0} = \frac{b}{1} = \frac{c}{2} = k \text{ (say)}$$

Which gives,

$a = 0,\ b = K,\ c = 2K$

Substituting the value of a,b,c in eqn (1)

$0 + K(y+2) + 2K(z-4) = 0$

$2y + 2z - 6 = 0$

which is required eqn. of plane

21. If from the point P(a,b,c) perpendiculars PL, PM be drawn to yz and zx-planes. Find the equation of the plane OLM.

⇒ Solution,

Give point $P(a,b,c)$

If PL, PM be perp. on yz and zx planes.

Then $L = (0,b,c)$ and $M = (a,0,c)$

The eqn. of plane passing through the origin is $Ax + By + Cz = 0 \longrightarrow (1)$

plane (1) passes through $(0,b,c), (a,0,c)$

So, $B \cdot b + C \cdot c = 0$ and

$A \cdot a + C \cdot c = 0$

Solving this by using cross-multiplication rule, we get

$$\frac{A}{bc} = \frac{B}{ca} = \frac{C}{-ab} = k \text{ (say)}$$

Which gives,

$A = bcK,\ B = caK,\ C = -abK$

Substituting the values of A,B,C in eqn. (1) we get

$bcKx + caKy + (-abK)z = 0$

$bcx + acy - abz = 0$
Which is required eqn. of plane.

22 The plane $lx + my = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. prove that the equation to the plane in its new position is $lx + my \pm \sqrt{l^2 + m^2} \tan\alpha \cdot z = 0$.

⇒ Solution,
The eqn. of plane through the intersection of planes $lx + my = 0$ and $z = 0$ is
$lx + my + Kz = 0 \longrightarrow (1)$
According to the question, α be the angle between planes $lx + my = 0$ and $lx + my + Kz = 0$,
Now,

$$\cos\alpha = \pm \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \pm \frac{l \cdot l + m \cdot m + 0}{\sqrt{l^2 + m^2 + 0}\sqrt{l^2 + m^2 + K^2}}$$

$$= \pm \frac{\sqrt{l^2 + m^2}}{\sqrt{l^2 + m^2 + K^2}}$$

Now,

$$\tan\alpha = \frac{\sqrt{1 - \cos^2\alpha}}{\cos\alpha} = \pm \frac{\sqrt{1 - \frac{l^2 + m^2}{l^2 + m^2 + K^2}}}{\sqrt{\frac{l^2 + m^2}{l^2 + m^2 + K^2}}}$$

$$= \pm \frac{\sqrt{l^2+m^2+k^2-l^2-m^2}}{\sqrt{l^2+m^2}}$$

$$= \pm \frac{\sqrt{k^2}}{\sqrt{l^2+m^2}}$$

$\therefore k = \pm \sqrt{l^2+m^2} \tan\alpha$

substituting the value of k in eq^n 1, we get,

$lx + my \pm \sqrt{l^2+m^2} \tan\alpha . z = 0$

proved

23 Find the bisectors of the angle bet^n two planes $2x-y+z=6$, $x+y+2z=3$.

⇒ Solution,

Given planes are,

$2x-y+z=6$ and $x+y+2z=3$

The eq^n of bisectors of angles are

$$\frac{2x-y+z-6}{\sqrt{4+1+1}} = \pm \frac{x+y+2z-3}{\sqrt{1+1+4}}$$

or, $2x-y+z-6 = \pm(x+y+2z-3)$

Now,

Taking -ve sign

$2x-y+z-6 = -x-y-2z+3$

$3x+3z-9=0$

or, $x+z-3=0 \longrightarrow (1)$

Taking +ve sign

$2x - y + z - 6 = x + y + 2z - 3$

$x - 2y - z - 3 = 0 \longrightarrow (2)$

Hence, Eq^n. (1) and (2) are the required bisectors of angle bet^n. planes.

24 Prove that the equation $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ represents a pair of perpendicular planes, find their equation.

$\Rightarrow$ Solution,

Given eq^n of plane is

$6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0 \longrightarrow (1)$

Comparing the eq^n (1) with general 2nd degree eq^n. $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

we have,

$a = 6, b = 4, c = -10, f = 3/2, g = 2, h = -11/2$

Same as Q.N. 13 - - - -