Tribhuvan University

2077

Bachelor Level (4 Yrs.)/ Science & Tech. /I Year

Analytical Geometry and Vector Analysis | Full Marks: 75

(MAT.102/ MATH -102) | Time: 3 hrs.

*Candidates are required to give their answers in their own words as far as practicable.*

*The figures in the margin indicate full marks.*

**MAT -102 (New Course):**

Attempt ALL the questions.

Group "A" 5×7=35

1. Define the auxiliary circle and the eccentric angle of a point in an ellipse. Find the point at which the line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Also find the condition for the line to be a normal to the ellipse. [5+2]
2. Find the nature and trace the conic represented by $36x^2 + 24xy + 29y^2 - 72x + 26y + 81 = 0$ [5+2]
3. How do you define a straight line mathematically? Find the equation of the line through (α, β, γ) parallel to the planes $lx + my + nz = p$ and $l'x + m'y + n'z = p'$. [1+6]

   OR

   What does the equation x + 4y + 3 = 0 represent in three dimensional geometry? Find the magnitude and the equation of the line of shortest distance between the lines:

   $\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1}$ and $\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$ [1+3+3]
4. What is a sphere? Find the equation of the sphere touching $x^2 + y^2 + z^2 - x + 3y + 2z = 3$ at the point (1, 1, -1) and passing through the origin [1+6]
5. Write the physical meaning of curl of a vector function of scalar variable. For any scalar function ϕ, show that

   $\text{curl}(\phi \vec{a}) = \phi(div\vec{a}) - \vec{a}(grad\phi)$ ,

   Also show that $\text{cur}(r^n \vec{r}) = 0$, where $\vec{r}$ denotes the position vector. [1+3+3]

   OR

   What does the magnitude of gradient of a vector function represent physically? Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2). [1+6]

## Group "B" 10×4=40

6. Translate the axes to change the equation $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ into an equation with linear terms missing. [4]

7. Show that the equation $8x^2 + 4xy + 5y^2 - 24x - 24y = 0$. Represents an ellipse. Hence find the equation of its major axis. [4]

OR

Show that the line $x\cos\alpha + y\sin\alpha = p$ will be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2\cos^2\alpha - b^2\sin^2\alpha = p^2$ [4]

8. Prove that the equation $\frac{1}{r} = 1 - e\cos\theta$ and $\frac{1}{r} = e\cos\theta - 1$ represent the same conic. [4]

9. Show that a second degree homogeneous equation in three variable represents a cone whose vertex is at origin.

OR

Find the equation of cylinder whose generators are parallel to the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and passing through the curve $x^2 + 2y^2 = 1, z = 0$. [4]

10. Find the equation of the planes which contain the line given by $5x + 6y - 18 = 0$ and $3y - z = 0$ and touch the ellipsoid $5x^2 + 3y^2 = 36$. [4]

11. Find the equation of the sphere which passes through the points (0, -2, -4), (2, -1, -1) and whose centre lies on the line $5y + 2z = 0 = 2x - 3y$. [4]

OR

Find the equation of sphere through the circle $x^2 + y^2 + z^2 = 1$, $2x + 4y + 5z = 6$ and touching the plane $z = 0$. [4]

12. Determine the equation of the cone with vertex (α, β, γ) and base $y^2 = 4ax, z = 0$. [4]

13. Prove the following :

a) $(\vec{a} \times \vec{b}) = a^2b^2 - (\vec{a}.\vec{b})^2$

b) if $\vec{a} + \vec{b} + \vec{c} = 0$ then $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{d}$ [4]

14. If $\vec{a}, \vec{b}, \vec{c}$ is the reciprocal system to the vectors $\vec{a}, \vec{b}, \vec{c}$ then prove that $\vec{a}'.\vec{a} = \vec{b}'.\vec{b} = \vec{c}'.\vec{c} = 1$ [4]

15. Prove that the necessary and sufficient condition for a vector function $\vec{a}$ of scalar variable t to have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$. [4]

OR

The acceleration of *a* moving particle at any time *t* is given by

$\frac{d^2\vec{r}}{dt^2} = 12\cos 2t\,\vec{i} - 8\sin 2t\,\vec{j} + 16t\,\vec{k}$. find the velocity function $\vec{v}$ and the displacement function $\vec{r}$ at any time *t*, provided that $\vec{v} = 0$ and $\vec{r} = 0$ when $t = 0$. [4]

□

**MATH 102 (Old Course):**

Attempt ALL the questions.

Group "A" 5×7=35

1. Define conic section. Find the centre, eccentricity, foci,latus rectum and length of the axes of the ellipse $x^2 + 4y^2 - 4x + 24y + 24 = 0$. [1+6]
2. State the condition under which the general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent (i) hyperbola (ii) ellipse. Find the centre of the conic section $2x^2 - 5xy - 3y^2 - x - 4y + 6 = 0$ unite its equation when transformed to the centre. [2+2+3]

   OR

   Define tangent and normal to a curve. Find the condition that any straight line $lx + my + n = 0$ may touch the conic.
   $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ [2+5]
3. Define skewlines and line of shortest distance. Find the shortest distance between the lines. $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+3}{2}$. Also find the equations of shortest distance. [1+1+5]
4. Define great and small circle of sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 0$ is a great circle. [2+5]

   OR

   Define tangent plant at a point of the sphere. Find the equation to the spheres which pass through the circle $x^2 + y^2 + z^2 = 5$ $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$ [1+6]
5. Define scalar triple product of three non -zero vectors. Interpret it geometrically. Also show that in the scalar triple product the position of dot and cross can be interchanged without clanging its value. [1+3+3]

Group "B" 10×4=40

6. What does the equation $2x^2 + y^2 - 4x + 4y = 0$ become when it is transferred to parallel axis through the point (1, -2)? [4]

7. If e and e' be the eccentricity of hyperbola and its conjugate then prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$ [4]

OR

Show that the line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $p^2 = a^2\cos^2\alpha + b^2\sin^2\alpha$. [4]

8. In any conic, prove that the sum of the reciprocals of the segments of any focal chord is constant. [4]

9. Find the equation of the plane through the interaction of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and origin. [4]

10. A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Prove that the locus of the centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$ [4]

11. Define reciprocal cone. Prove that the cones $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ are reciprocal. [1+3]

OR

Obtain the equation of the cylinder which passes through $y^2 = 4ax$, $z = 0$ and whose generators are parallel to the line $x = y = z$ [4]

12. Show that the plane $14x + 5y + 9z = 60$ touches the ellipsoid $7x^2 + 5y^2 + 3z^2 = 60$ and find the point of contact. [4]

OR

Show that the locus of the centres of sections of a central conicoid which passes through a given line is a conic. [4]

13. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors then prove that $\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' = 3$ [4]

14. Prove that the necessary and sufficient condition for a vector function $\vec{a}$ of scalar variable $t$ to have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$. [4]

15. Define curl of a vector function If $\theta$ is a scalar function, prove that curl $(\text{grad}\theta) = 0$ [1+3]

OR

Define divergence of a vector function. If $f = x^3 + y^3 + z^3 - 3xyz$ then find div(grad $f$). [1+3]