unit-1

Transformation of coordinates

→ 4 marks (1 short question)

1. Find the equation of the curve $9x^2 + 4y^2 + 18x - 16y = 11$ referred to parallel axis ~~theorem~~ through $(-1, 2)$.
[2062, 2070, 2072]

⇒ Solution,

Given equation of curve is

$9x^2 + 4y^2 + 18x - 16y = 11$ → (1)

When the origin is shifted to ~~the~~ $(-1, 2)$ and axes are parallel to the original axes, then

New co-ordinates are (X, Y) such that

$x = X + h = X - 1$

$y = Y + K = Y + 2$

Substituting these values of x and y in eqn. (1), we get

$9(X-1)^2 + 4(Y+2)^2 + 18(X-1) - 16(Y+2) = 11$

or, $9X^2 - 18X + 9 + 4Y^2 + 16Y + 16 + 18X - 18 - 16Y - 32 - 11 = 0$

or, $9X^2 + 4Y^2 - 36 = 0$. #

Which is the required eqn. of curve.

2. What does the the equation $x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2$ become when the axes are turned through an angle of $30°$ to the original axis.
[2054, 2057, 2058, 2060, 2071 new]

⇒ Solution,

Given eqⁿ is

$x^2 + 2\sqrt{3}xy - y^2 = 2a^2 \longrightarrow (1)$

Let (X, Y) be the new co-ordinates axes and (x, y) be the original axes, After the rotating the new axes by an angle $30°$. Then

$$x = X\cos\theta - Y\sin\theta$$
$$= X\cos 30° - Y\sin 30°$$
$$= X\cdot\frac{\sqrt{3}}{2} - Y\frac{1}{2}$$
$$= \frac{1}{2}(\sqrt{3}X - Y)$$

and, $y = X\sin\theta + Y\cos\theta$
$$= X\sin 30° + Y\cos 30°$$
$$= X\frac{1}{2} + Y\frac{\sqrt{3}}{2}$$
$$= \frac{1}{2}(X + \sqrt{3}Y)$$

Substituting this values of x and y in eqⁿ (1), we get

$$\left[\frac{1}{2}(\sqrt{3}X - Y)\right]^2 + 2\sqrt{3}\cdot\frac{1}{2}(\sqrt{3}X - Y)(X + \sqrt{3}Y)\cdot\frac{1}{2} - \left[\frac{1}{2}(X + \sqrt{3}Y)\right]^2 = 2a^2$$

or, $\frac{1}{4}\left[3x^2-2\sqrt{3}xy+y^2\right]+\frac{2\sqrt{3}}{4}\left[\sqrt{3}x^2+3xy-xy-\sqrt{3}y^2\right]-\frac{1}{4}\left(x^2+2\sqrt{3}xy+3y^2\right)$

$=2a^2$

or, $3x^2-2\sqrt{3}xy+y^2+2\times3x^2+6\sqrt{3}xy-2\sqrt{3}xy-6y^2-x^2-2\sqrt{3}xy-3y^2=8a^2$

or, $8x^2-8y^2-8a^2=0$

or, $x^2-y^2-a^2=0$

Which is required eq^n. #

3. Transform the following equation to the axis inclined $45°$ to the origin axes $y^4+x^4+6x^2y^2=2$

⇒ Solution,

Given eq^n is

$y^4+x^4+6x^2y^2=2 \longrightarrow (1)$

Let (X, Y) be the new co-ordinates axes when the axes are rotated by angle $\theta=30°$

Then,

$x = X\cos\theta - Y\sin\theta$

$= X\cos45° - Y\sin45°$

$= X\cdot\frac{1}{\sqrt{2}} - Y\cdot\frac{1}{\sqrt{2}}$

$$= \frac{1}{\sqrt{2}}(x-y)$$

and

$$y = x\sin\theta + y\cos\theta$$
$$= x\sin 45^\circ + y\cos 45^\circ$$
$$= x\cdot\frac{1}{\sqrt{2}} + y\cdot\frac{1}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}(x+y)$$

Now, substituting these values in eq^{n} (1), we get

$$\Rightarrow \left[\frac{1}{\sqrt{2}}(x+y)\right]^4 + \left[\frac{1}{\sqrt{2}}(x-y)\right]^4 + 6\left[\left(\frac{1}{\sqrt{2}}(x-y)\right)^2\left(\frac{1}{\sqrt{2}}(x+y)\right)^2\right] = 2$$

or, $\frac{1}{4}(x+y)^4 + \frac{1}{4}(x-y)^4 + \frac{6}{4}\left[(x-y)^2(x+y)^2\right] = 2$

or, $(x^2+2xy+y^2)^2 + (x^2-2xy+y^2)^2 + 6\left[(x-y)(x+y)\right]^2 = 8$

or, $x^4 + 4x^2y^2 + y^4 + \cancel{4x^3y} + 2x^2y^2 + \cancel{4xy^3} + x^4 + 4x^2y^2 + y^4 - \cancel{4x^3y} + 2x^2y^2 - \cancel{4xy^3} + 6(x^2-y^2)^2 = 8$

or, $2x^4 + 2y^4 + \cancel{12x^2y^2} + 6x^4 + 6y^4 - \cancel{12x^2y^2} = 8$

Date ____
Page ____

or, $8X^4 + 8Y^4 = 8$

or, $X^4 + Y^4 = 1$
Which is required equation.

4. If the axes be turned through an angle $\tan^{-1}(2)$ what does the equation $4xy - 3x^2 = a^2$ become? [2054, 2068 N]

⇒ Solution,
Given equation is
$4xy - 3x^2 = a^2 \longrightarrow (1)$
Let (X, Y) be the new co-ordinates when the axes are rotated by angle $\theta = \tan^{-1}(2)$ Then
$x = X\cos\theta - Y\sin\theta$
$y = X\sin\theta + Y\cos\theta$

$\theta = \tan^{-1}2$
or, $\tan\theta = 2$

So, $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+2^2}}$

$= \frac{1}{\sqrt{5}}$

and $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{5}} = \sqrt{4/5}$

$= \frac{2}{\sqrt{5}}$

Then,

$$x = X\frac{1}{\sqrt5} - Y\frac{2}{\sqrt5}$$

$$= \frac{1}{\sqrt5}(X-2Y)$$

and, $y = X\frac{2}{\sqrt5} + Y\frac{1}{\sqrt5}$

$$= \frac{1}{\sqrt5}(2X+Y)$$

Substituting these value of x and y in eqn (1), we get

$$4\left[\frac{1}{\sqrt5}(X-2Y)\cdot\frac{1}{\sqrt5}(2X+Y)\right] - 3\left[\frac{1}{\sqrt5}(X-2Y)\right]^2 = a^2$$

or, $4\left[\frac{1}{5}(2X^2+XY-4XY-2Y^2)\right] - \frac{3}{5}(X^2-4XY + 4Y^2) = a^2$

or, $\frac{1}{5}\left[8X^2+4XY-16XY-8Y^2-3X^2+12XY-12Y^2\right] = a^2$

or, $5X^2 - 20Y^2 = 5a^2$

or, $X^2 - 4Y^2 = a^2$

Which is required equation.

5. If the axes be turned through an angle $\tan^{-1} 3$ what does the equation $3xy - 4y^2 = a^2$ becomes? [2070]

⇒ solution,

Given eqn is

$3xy - 4y^2 = a^2 \longrightarrow (1)$

Let (X, Y) be the new co-ordinates when the axes are rotated by $\theta = \tan^{-1} 3$

or, $\tan\theta = 3$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}}$$

$$= \frac{1}{\sqrt{1+3^2}}$$

$$= \frac{1}{\sqrt{10}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1 - \frac{1}{10}}$$

$$= \sqrt{\frac{10-1}{10}}$$

$$= \frac{3}{\sqrt{10}}$$

Then,

$x = X\cos\theta - Y\sin\theta$

$= X \cdot \frac{1}{\sqrt{10}} \quad Y \cdot \frac{3}{\sqrt{10}}$

$$= \frac{1}{\sqrt{10}}(x-3y)$$

and

$$y = x\sin\theta + y\cos\theta$$

$$= x\cdot\frac{3}{\sqrt{10}} + y\,\frac{1}{\sqrt{10}}$$

$$= \frac{1}{\sqrt{10}}(3x+y)$$

Substituting these values of x and y in eqn (1), we get

$$3\left[\frac{1}{\sqrt{10}}(x-3y)\cdot\frac{1}{\sqrt{10}}(3x+y)\right] - 4\left[\frac{1}{\sqrt{10}}(3x+y)\right]^2 = a^2$$

or, $\frac{3}{10}\left[3x^2+xy-9xy-3y^2\right] - \frac{4}{10}\left[9x^2+6xy+y^2\right] = a^2$

or, $\frac{1}{10}\left[9x^2+3xy-27xy-9y^2-36x^2-24xy-4y^2\right] = a^2$

or, $-27x^2-13y^2-48xy = 10a^2$

or, $27x^2+48xy+13y^2+10a^2 = 0$ #

6. What does the equation $(a-b)(x^2+y^2) - 2abx = 0$ become if the origin be moved to the point $\left(\frac{ab}{a-b}, 0\right)$?

⇒ Solution,

Given eqn. is [2061, 2064]

$(a-b)(x^2+y^2) - 2abx = 0 \longrightarrow (1)$

Let (X, Y) be the new coordinates when origin be moved to $\left(\frac{ab}{a-b}, 0\right)$

Then,

$$x = X + \frac{ab}{a-b}$$

$$y = Y + 0 = Y$$

Substituting these values of x, y in eqn. (1), we get

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)^2 + Y^2\right] - 2ab\left(X + \frac{ab}{a-b}\right) = 0$$

or, $(a-b)\left[X^2 + 2X\cdot\frac{ab}{a-b} + \frac{a^2b^2}{(a-b)^2} + Y^2\right] - 2abX - \frac{2a^2b^2}{a-b} = 0$

or, $(a-b)X^2 + \cancel{2X}ab + \frac{a^2b^2}{(a-b)} + (a-b)Y^2 - 2a\cancel{b}X - \frac{2a^2b^2}{a-b} = 0$

or, $(a-b)x^2 - \frac{a^2b^2}{a-b} + (a-b)y^2 = 0$

or, $(a-b)^2x^2 - (a-b)^2y^2 - a^2b^2 = 0$

Which is required eq^n.

7 What does the equation $(x-h)^2 + (y-k)^2 = r^2$ become when it is transferred to parallel axes through the point $(h, k-r)$? [2061]

⇒ Solution,

Given eq^n is

$(x-h)^2 + (y-k)^2 = r^2 \longrightarrow (1)$

Let (X, Y) be the new co-ordinates when it transfered to parallel axes through $(h, k-r)$, Then

$x = X + h$

$y = Y + k - r$

Substituting these values in eq^n (1), we get

$\therefore [(X+h) - h]^2 + [Y + k - r - k]^2 = r^2$

or, $X^2 + Y^2 - 2Yr + r^2 = r^2$

or, $X^2 + Y^2 - 2rY = 0$

Which is required equation.

8. What does the equation $(x-a)^2+(y-b)^2=c^2$ become when it is transferred to parallel axes through the point $(a-c, b)$? [2055, 66]

⇒ Solution,

Given eq^n is

$(x-a)^2 + (y-b)^2 = c^2$ ——→ (1)

Let (X, Y) be the new co-ordinates axes when it transferred to parallel axes through the point $(a-c, b)$

Then,

$x = X + a - c$

$y = Y + b$

Substituting these values in eq^n (1), we get

$(X+a-c-a)^2 + (Y+b-b)^2 = c^2$

or, $X^2 + c^2 - 2Xc + Y^2 = c^2$

or, $X^2 + Y^2 - 2cX = 0$

Which is required eq^n.

9. If the axes be turned through an angle α what does the equation $x^2 + 2xy\tan 2\alpha - y^2 = a^2$ become?

⇒ Solution,

Given equation is

$x^2 + 2xy\tan^2\alpha - y^2 = a^2 \longrightarrow (1)$

Let (X, Y) be the new co-ordinates when it rotating by an angle α. Then

$x = X\cos\alpha - Y\sin\alpha$ and

$y = X\sin\alpha + Y\cos\alpha$

Substituting this values in eq^n (1), we get

$(X\cos\alpha - Y\sin\alpha)^2 + 2(X\cos\alpha - Y\sin\alpha)(X\sin\alpha + Y\cos\alpha)\tan^2\alpha - (X\sin\alpha + Y\cos\alpha)^2 = a^2$

or, $X^2\cos^2\alpha - 2XY\cos\alpha\sin\alpha + Y^2\sin^2\alpha + 2\tan^2\alpha(X^2\cos\alpha\sin\alpha + XY\cos^2\alpha - XY\sin^2\alpha - Y^2\cos\alpha.\sin\alpha) - X^2\sin^2\alpha + 2XY\sin\alpha\cos\alpha - Y^2\cos^2\alpha = a^2$

or, $X^2(\cos^2\alpha - \sin^2\alpha) + Y^2(\sin^2\alpha - \cos^2\alpha) - 4\sin\alpha.\cos\alpha XY + 2\tan^2\alpha[(\cos^2\alpha - \sin^2\alpha)XY + (X^2 - Y^2)\sin\alpha\cos\alpha] = a^2$

or, $(X^2 - Y^2)(\sin 2\alpha . \tan^2\alpha + \cos 2\alpha) + 2XY(\cos 2\alpha \tan^2\alpha - \sin 2\alpha) = a^2$

Which is required eq^n.

10. What does the equation $2x^2+y^2-4x+4y=0$ become when it is transferred to parallel axes through the point $(1,-2)$? [2067]

$\Rightarrow$ Solution,

Given equation is

$2x^2+y^2-4x+4y=0 \longrightarrow (1)$

Let (X,Y) be the new co-ordinates when it transferred to parallel axes through the point $(1,-2)$. Then

$x = X+1$

$y = Y-2$

Substituting these values of x and y in eqn (1), we get

$2(X+1)^2+(Y-2)^2-4(X+1)+4(Y-2)=0$

or, $2X^2+\cancel{4X}+2+Y^2-\cancel{4Y}+4-\cancel{4X}-4+\cancel{4Y}-8=0$

or, $2X^2+Y^2-6=0$

which is the required equation.

11. Translate the axes to change $3x^2-2xy+4y^2+8x-10y+8=0$ into an equation with linear term missing.

$\Rightarrow$ Solution,

Given eqn. is

$3x^2-2xy+4y^2+8x-10y+8=0 \longrightarrow (1)$

Let the origin be shifted to the point (h, k). Then transformed eq^n is obtained by putting

$x = X + h$ & $y = Y + k$

Then eq^n (1) becomes

$$3(X+h)^2 - 2(X+h)(Y+k) + 4(Y+k)^2 + 8(X+h) - 10(Y+k) + 8 = 0$$

or, $3X^2 + 6hX + h^2 - 2XY - 2Xk - 2hY - 2hk + 4Y^2 + 8kY + k^2 + 8X + 8h - 10Y - 10k + 8 = 0$

or, $3X^2 + 4Y^2 - 2XY + X(6h - 2k + 8) + Y(-2h + 8k - 10) + (3h^2 - 2hk + 4k^2 + 8h - 10k + 8) = 0$ (2)

We have,

For linear term missing, Coeff. of X and coeff. of Y must be zero in above eq^n.

Then

$6h - 2k + 8 = 0$

or, $3h - k = -4 \longrightarrow$ (I)

and

$-2h + 8k - 10 = 0$

or, $-h + 4k = 5 \longrightarrow$ (II)

Solving eq^n (I) and (II), we get

$h = -1 \quad k = 1$

Now,

Substituting these values of (h, k) in

Page-15

eq^n (2) we get

$3x^2+4y^2-2xy+x(-6-2+8)+y(2+8-10)$
$+(3+2+4-8-10+8)$

or, $3x^2+4y^2-2xy-1=0$

or, $3x^2-2xy+4y^2=1$

which is required eq^n.