Tribhuvan University

2078 (Partial) / 2079 (Regular)

Bachelor Level (4 Yrs.)/ Science & Tech. /I Year

(MAT - 101) : Calculus — Full Marks: 75

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

(New Course)

Attempt ALL the questions.

Group "A" — 5×7=35

1. State the theorem for n^{th} derivative of the product of two function. If $y = \sin(\text{sing } x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$. Also, find the n^{th} derivative of $y = x\tan^{-1} x$. [1+2+4]

   OR

   State Cauchy's Mean Value theorem. When does it become Lagrange's Mean Value theorem? Verify Cauchy's mean Value theorem if f(x) = cosx, g(x) = sinx on the interval [a, b]. [1+2+4]

2. Define the term 'chord of curvature'. Show that the chord of curvature through the pole of the curve $r^m = a^m \cos m\theta$ is $\frac{2r}{m+1}$. [1+6]

3. Define Eulerian first and second integrals. Prove that [1+3+5]

   (i) $B(m, n) = 2\int_0^{\pi/2} \sin^{2m-1} x \cos^{2n-1} x\, dx$

   (ii) $\int_0^{\infty} e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$

4. State Green's theorem in plane. Evaluate $\oint_C y^2dx + 3xy\,dy$, where C is the boundary of the semi angular region D in the upper half-plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. 3.5 [2+5]

OR

Define curl and divergence of a vector field. If the vector field $F(x, y, z) = xz\vec{i} + xyz\vec{j} - y^2\vec{k}$, then find curl F. Also, show that this vector field F is not conservative. [2+3+2]

5. Define a linear differential equation of first order. Explain the method of solving such an equation. Reduce the equation $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$ to the linear form and then solve it. [1+1+5]

Group "B" 10×4=40

6. Find the angle between the two curves $x^2 + y^2 + 4x=16$, and $y^2 = 2x$. [1+3]

OR

Show that the sum of the intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant. [4]

7. Define an asymptote to curve. Find all asymptotes to the curve $y^2 - x^2 - 2x - 2y - 3 = 0$. [1+3]

8. If $u = z\tan^{-1}\frac{x}{y}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. [4]

OR

Find the extreme value of the function $x^2 + y^2$ under the condition $x + 4y = 2$ [4]

9. Find a parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$ [4]

10. Evaluate $\iint_S \vec{r}.\vec{n}dS$ where S is a closed surface enclosing a volume V. [4]

11. Find from definition the value of $\int_0^2 x^2\,dx$. [4]

4 12. Evaluate the iterated integrals $\int_0^3 \int_1^2 x^2 y \, dy \, dx$. [4]

13. Find the area included between the two parabolas $y^2=4ax$ and $x^2=4ay$.

OR [4]

4 Find the volume of the solid generated by revolving the asteroid $x^{2/3}+y^{2/3}=a^{2/3}$ about the axis of x.

14. What do you mean by the initial condition? Solve $\frac{dy}{dx}=x-y$ given that y = 2 when x = 0. [1+3]

15. Prove that $\frac{1}{f(D)}(e^{ax}V)=e^{ax}\frac{1}{f(D+a)}V$. [4]

OR

Solve $(D^2+4D+3)y=e^{-3x}$. Where $D=\frac{d}{dx}$.

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(Old Course)

Attempt ALL the questions.

Group "A" 5×7=35

1. State the theorem for n^{th} derivative of the product of two function. If y= sin (sing x), prove that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0$. Also, find the n^{th} derivative of $y=x^3e^{bx}$. [1+2+4]

OR

State Lagrange's Mean Value theorem. Give its geometrical meaning. Verify Lagrange's Mean Value theorem for $f(x)=(x-1)(x-2)(x-3)$ on the interval [1,4] [1+2+4]

2. Define the term 'chord of curvature'. Show that the chord of curvature through the pole of the curve $r^m = a^m \cos m\theta$ is $\frac{2r}{m+1}$. [1+6]

3. Find the reduction formula for $\int \cos^n x\,dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \cos^4 x\,dx$. [5+2]

OR

Define Eulerian first and second integrals. Prove that

(i) $B(m, n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$

(ii) $\Gamma\frac{3}{2} = \frac{\sqrt{\pi}}{2}$ [2+3+2]

4. Define polar subtangent and polar subnormal and deduce expression at any point $P(r, \theta)$ of the curve $r = f(\theta)$. [2+5]

5. Define a linear equation of first order. Explain the method of solving such an equation. Reduce the equation $\frac{dy}{dx} + \frac{y}{x} = y^2$ to the linear form and then solve it. [1+1+5]

Group "B" 10×4=40

6. Find the pedal equation of the curve $y^2 = 4a(x + a)$. [4]

P.T.O.

7. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$. [4]

OR

If the limit of $\frac{\sin 2x + a\sin x}{x^3}$ as x tends to zero, be zero, find value of a. [4]

8. Verify Euler's theorem for the function $u = x^n \tan^{-1}\frac{y}{x}$. [4]

9. Evaluate : $\int_0^{\frac{\pi}{2}}\frac{dx}{1+\sqrt{\tan x}}dx.$ [4]

OR

Prove that [4]

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 & when\ f(x)\ is\ an\ odd\ function \\ 2\int_0^a f(x)dx & when\ f(x)\ is\ an\ even\ function \end{cases}$$

10. Find, from the definition, $\int_0^2 x^2 dx$. [4]

11. Trace the curve $xy^2 = 4a\,(2a - x)$. [4]

OR

Find all asymptotes to the curve $y^2 - x^2 - 2x - 2y - 3 = 0$ [4]

12. Evaluate $\int_0^2\int_1^2 x^2 y\,dy\,dx.$ [4]

13. Prove that $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$ when $a \neq 0$, where $D = \frac{d}{dx}$. [4]

OR

Write a homogeneous linear differential equation of the second order in standard form. How does the equation

$(a+bx)^2 \frac{d^2y}{dx^2} + P_1(a+bx)\frac{dy}{dx} + P_2 y = Q$ reduce to the homogeneous linear form? [1+3]

14. Find the general and singular solution of $y = px + ap(1-p)$, where $p = \frac{dy}{dx}$. [4]

15. Solve $(D^2 + 9)y = \cos x$, Where $D = \frac{d}{dx}$. [4]

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