Tribhuvan University

2082 (Regular)

Bachelor Level (4 Yrs.)/ Science & Tech. /I Year

Calculus — Full Marks: 75

(MAT - 101) — Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt ALL the questions.

Group "A" — 5×7=35

1 State Lagrange's theorem for two variables u and v where u and v are functions of x. if $y = \tan^{-1} x$, then prove that

a) $(1+x^2)y_1 = 1$,

b) $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$. [1+2+4]

OR

State Lagrange's Mean Value Theorem. Verify the Lagrange's Mean Value Theorem of the function $f(x) = x(x-1)$ for [0, 0.5] [1+6]

2. State the fundamental theorem of Calculus with an example.

Evaluate $\int_a^b \frac{1}{x} dx$ as a limit of sum. [3+4]

OR

Define Beta and Gamma function. prove that

a) $\Gamma(n+1) = n\Gamma(n)$

b) $\int_{-1}^{1} (1-x^2)^{\frac{2}{3}} dx = \beta\left(\frac{1}{2}, \frac{5}{3}\right)$. [2+1+4]

3 How do you define the maximum and minimum values of the functions of two variables? Find the extreme values of xy^2 when $x+y=1$. [2+5]

4 What do you mean by the curvature and radius of curvature of a curve? Show that the circle is a curve of uniform curvature and its radius of curvature at every points is constant. [2+1+4]

5. Define a linear differential equation of first order. Explain the method of solving such equation. Reduce the equation $\frac{dy}{dx}+2xy=x^3y^3$ to linear form and hence solve it. [1+2+4]

Group "B" 10×4=40

6. What is the pedal equation of a curve? Deduce its equation from Cartesian equation. [1+3]

7. Write down any seven indeterminate forms. Prove that $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}$. [2+2]

8. Trace the curve $y^2=(x-2a)^3$. [4]

OR

Find all asymptotes of the cubic $x^3-2y^3+xy(2x-y)+y(x-y)+1=0$. [4]

9. If $z=\frac{\cos y}{x}$ and $x=u^2-v,\ y=e^v$, then find $\frac{\partial z}{\partial v}$. [4]

OR

If $u=\cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\cot u=0$. [4]

10. Find the volume of the solid formed by revolution of the curve $y^2=x^2(a-x)$ about x-axis. [4]

11. Find the total work done in a moving a particle in a force field given by $\vec{F}=2xy\vec{i}-3x\vec{j}-5z\vec{k}$ along the curve $x=t, y=t^2+1, z=2t^2$ from $t=0$ to $t=1$. [4]

12. Use Stoke's theorem to evaluate $\int_C \vec{F}.dr$, where $\vec{F}(x,y,z)=(x+y^2)\vec{i}+(y+z^2)\vec{j}+(z+z^2)\vec{k}$, where C is the triangle with vertices (1,0,0), (0,1,0) and (0, 0, 1) [4]

OR

State Stoke's theorem and use it to deduce $\int_C \vec{r}\,d\vec{r}=0$ [4]

13. Find the particular integral (P.I.) of the differential equation

$$\frac{d^2y}{dx^2} + P_1\frac{dy}{dx} + P_2 y = e^{ax}$$. Also, find the P.I. of the equation $(D - 2)^2 y = 6e^{2x}$. [2+2]

OR

Solve $(D^2 + 3D + 2)y = e^{2x}$, given that $y = 1$, $\frac{dy}{dx} = 0$ when $x = 0$. [4]

14. Evaluate $\int_0^3 \int_1^2 x^2 y \, dy \, dx$. [4]

15. Determine the order and degree of the differential equation with justification (if exist)

(i) $\left(\frac{d^2y}{dx^2}\right)^2 + 3\left(\frac{dy}{dx}\right)^4 + y = 0$ (ii) $\frac{dy}{dx} + \cos\left(\frac{dy}{dx}\right) = 0$ [2+2]