Tribhuvan University

2081(Regular)

Bachelor Level (**4 Yrs.**)/ Science & Tech. /II Year

Linear Algebra

(Mat 201)

Full Marks: 75

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt ALL the questions.

Group "A" **5×8=40**

1. Write the difference between the row echelon and the row reduced echelon form of a matrix. Find the row echelon form and row reduced echelon form of the matrix : $\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$ [2+3+3]

2. Define linear combination following vectors in $\mathbb{R}^n$, Is the vector $\vec{w} = (-1, 3, 7)$ a linear combination of the vectors $\vec{u} = (4, 2, 7)$, and $\vec{v} = (3, 1, 4)$? Define the span of a set of vectors. Let $\vec{a} = \begin{pmatrix} 10 \\ 11 \\ 4 \end{pmatrix}$. $\vec{v}_1 = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$. Is the vector $\vec{a}$ in the span of vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$? If so, express vector $\vec{a}$ as a linear combination of vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$. [1+3+3+1]

3. Let L_1 and L_2 be two line in $\mathbb{R}^n$ described as $L_1 = \{u + t\,v : t \in \mathbb{R}\}$, $L_2 = \{w + s\,z : s \in \mathbb{R}\}$. Prove that these lines are the same if and only if $u - w$ and v are multiple of z. Are these two lines $L_1 = (-3, -3, -6) + t\,(9, 15, 12)$, and $L_2 = (3, 7, 2) + s\,((3, 5, 4)$ same ? Explain.

4. Describe the effects of the linear transformation of the following with the figure:

(a) The linear mapping that has the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(b) The linear mapping that has the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ [4+4]

OR

Define vector space, linearly dependent and linearly independent sets of vectors in vector space. Determine whether the set of vectors $\{u_1, u_2, u_3\}$, where $u_1 = (2, -3, 1)$, $u_2 = (3, -5, 2)$, $u_3 = (4, -5, 1)$, are linearly dependent or independent. [2+1+1+4]

5. Define vector subspace and inverse image. Prove that if f is a linear transformation from a vector space X to a vector space Y, then for any subspace V in Y, $f^{-1}(V)$ is a subspace of X. Let $f : \mathbb{R}^4 \to \mathbb{R}^3$ and is defined by $f(x_1, x_2, x_3, x_4) = (3x_1, 2x_2 - x_1, x_4)$ and let, $V = Span\{z, w\}$, where $z = (1, 1, 0)$, $w = (0, 1, 1)$. Describe the set $f^{-1}(V)$ in simple terms. [1+1+3+3]

OR

Define the basis of a vector space. Show that a set of vectors $\{(1, 1, 1), (0, 1, 0), (1, 2, 3)\}$ form a basis for $\mathbb{R}^3$. [2+6]

Group "B" 5×7=35

6. Define the upper triangular matrix with an example. Prove that the product of two upper triangular matrices of the same order is also an upper triangular matrix. Verify it by taking one example. [1+1+4+1]

P.T.O.

7. For square matrices of the same size, prove $Det\,(AB) = Det\,(A).\ Det(B)$.

Verify this result when $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 4 \\ 1 & 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 1 & 3 \\ 5 & 1 & 4 \end{bmatrix}$ [4+3]

8. Prove that any set of eigenvectors corresponding to distinct eigenvalues of a matrix is linearly independent. Find the eigenvalue and corresponding eigenvector of a matrix $\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$. [4+3]

9. Define orthogonal complement. Determine whether the vector $x = (3, 7, 1)$ in the orthogonal complement of the pair of vectors $\{u, v\}$, where $u = (5, -2, -1)$, and $v = (2, -3, 15)$? In an inner product space, prove that the orthogonal complement of any subset is a subspace. [1+2+4]

OR

Apply the Gram-Schmidt process to find orthogonal basis and orthonormal basis vectors for $\mathbb{R}^3$ to the vectors $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$, $u_3 = (0, 0, 1)$. [7]

10. Define self-adjoint mapping. Prove that all eigenvalues of a self-adjoint operator are real. And also prove that any two eigenvectors of a self-adjoint operator if associated with different eigenvalues, are mutually orthogonal. [1+3+3]

OR

State Cayley-Hamilton theorem. verify this theorem for matrix

$A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$. [1+6]