Tribhuvan University

2081(Regular)

Bachelor Level (4 Yrs.)/ Science & Tech. /II Year

Differential Equation

(MAT 202)

Full Marks: 75

Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt ALL the questions.

Group "A"

5×8=40

1. (a) Define nth order ordinary differential equation with examples. [2]

(b) Verify $y = 3t + t^2$ is a solution of differential equation $t\,y' - y = t^2$ [2]

(c) Consider the differential equation $\frac{dy}{dt} = -ay + b$, where both a and b are positive real numbers

i) Solve the differential equation.

ii) Sketch the solution for several differential initial conditions. [2+2]

2 (a) Solve the initial value problem $y' = \frac{x(x^2+1)}{4y^3}, y(0) = -\frac{1}{\sqrt{2}}$ in explicit form [4]

(b) consider a tank used in certain hydrodynamic experiments. After one experiment, the tank contains 150 L of a dye solution with a concentration of 1 g/L. To prepare for next experiment the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well - stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value. [4]

OR

(a) Find an integrating factor and solve the given equation $dx + \left(\frac{x}{y} - \sin y\right)dy = 0$. [4]

(b) An investor deposits $ 1000 in an account paying interest at the rate of 7% compounded monthly and also makes additional deposits of $25 per month. Find the balance in the account after 4 years. [4]

3. Find the general solution of the given initial value problem.
$y'' + 4y = 2\sin 2t,\ y(0) = 2,\ y'(0) = -1,$ [8]

OR

A spring is stretched by 10 cm by a force of 2 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 4 N when the velocity is 1 m/s. The motion of the mass is driven by an external force of $4\cos 2t$ N. If the mass is initially at rest at equilibrium, find its position at any time t. Identify the transient and steady state parts of the solution. [8]

4. Find the general solution of the given IVP $y'' + y' - 2y = 2t$, $y(0) = 0$, $y'(0) = 1$ [8]

5 Consider the system $x_1^1 = x_1 - 2x_2, x_1(0) = -1$

$x_2^1 = 3x_1 - 4x_2,\ x_2(0) = 2.$

a) Transform the above system into a single equation of second order.
b) Find x_1 and x_2 that also satisfy the given initial conditions. [4+4]

Group "B" 5×7=35

6. (a) How do you solve the Clairaut's differential equation $y = px + f(p)$ where $p = \frac{dy}{dx}$ [3]

(b) Find the complete and singular solution of the equation
$y = px + ap(1-p)$ [7]

7. Find the surface which is orthogonal to the one-parameter system. $Z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2, Z = 0.$ [7]

8. Solve $p^2x + q^2y = Z$ using Charpit's method. [7]

P.T.O.

9. Sketch the graph of the function for three periods. Find the Fourier series for the given function

$$f(x) = \begin{cases} x + L, & -L \le x \le 0, \\ L, & 0 < x < L, \end{cases} \qquad f(x + 2L) = f(x) \qquad [7]$$

OR

Find the eigenvalues and eigenfunctions of the given boundary value problem. Assume that all eigenvalues are real $y'' + \lambda y = 0, y'(0) = 0,$ $y(\pi) = 0.$ [7]

10. Find the solution $u(x, y)$ of two dimensional Laplace's equation $u_{xx} + u_{yy} = 0$ in the rectangle $0 < x < a,\ 0 < y < b$ that satisfies the boundary conditions

$$u(0, y) = 0,\ u(a, y) = 0;\ 0 < y < b$$

$$u(x, 0) = h(x),\ u(x, b) = 0;\ 0 \le x \le a, \qquad [7]$$

OR

Find the solution $u(r, \theta)$ of Laplace equation in semi-circular region $r < a,\ 0 < \theta < \pi$ that satisfies the boundary conditions.

$$u(r, 0) = 0,\ u(r, \pi) = 0,\ 0 \le r < a$$

$$u(a, \theta) = f(\theta),\ 0 \le \theta \le \pi$$

by assuming that u is single valued and bounded in the region. [7]

□